International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Studying the properties of a superpotential using algebraic equations

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The real world as we know it occurs at energies well below the Planck scale, so it is very well described by effective field theory. These effective field theories arise as low-energy descriptions of some "vacuums" of string theory, which in some approximate schemes can be considered as solutions of the equations of motion for a compactification space. In attempts to understand the fundamentals of string theory, it has become clear that we need a better understanding of conformal theories as these are the building blocks of string vacua. Conformal theories are in general very complicated but using the renormalization group (RG) theory and the identification of fixed points of RG flow with conformal theories, we can characterize the conformal theory by the corresponding data. This approach is most powerful when applied to superconformal models with N = 2 worldsheet supersymmetry [1]. The action for a $N = 2$ supersymmetric quantum field theory takes the form

$$\int d^2zd^4\theta K(\overline{\Phi_i}, \Phi_i) + \left(\int d^2zd^2\theta W(\Phi_i) + \ c.c.\right),$$

where W is a holomorphic function of the chiral superfields Φ_i. W is not renormalized and provides us with an invariant of the renormalization group flow with which to characterize two-dimensional theories. For example, the Landau-Ginzburg super-potential $W(\Phi) = \Phi^{p+2}$ corresponds to the A-series modular invariant $N = 2$ minimal theory of level p and central charge $c = 3p/(p + 2)$. For a tensor product of minimal models we have a superpotential

$$W(\Phi_1, \ldots, \Phi_r) = \Phi_1^{p_1+2} + \ldots + \Phi_r^{p_r+2}.$$

At the fixed point of superpotential, the theory must be scale invariant, and so potential has the property that if one scales the fields according to

$$\Phi_i \to \lambda^{\omega_i}\Phi_i,$$

then the potential scales by

$$W(\lambda^{\omega_i}\Phi_i) = \lambda W(\Phi_i).$$

Such functionals are called quasi-homogeneous. The scale invariance is connected with conformal field theory. In particular for modal deformations to be considered as physical moduli of the conformal field theory, they should respect the quasi-homogeneity of the superpotential. This is a special property of $N = 2$ theories, and follows from the non-renormalization theorems. These superpotentials could be shown by checking the correspondence between the central charge c, the dimension of chiral fields, and the ring of the corresponding minimal model. This means that we can obtain the Calabi-Yau manifold with the tensor product of the minimal discrete models from the point of view of L G theory [2].

We considerd different $N = 2, 3, 4$ models, calculated corresponding central charges, $c = 3, 6, 9$ and investigated the forms and roots of such manifolds for singular 2-fold, or K3 surface, defined by the following polynomials [3]

$$F_{A_{N-1}} = x_1^N + x_2^2 + x_3^2, (N \geq 2)$$

$$F_{E_6} = x_1^4 + x_2^3 + x_3^2$$
$$F_{E_8} = x_1^5 + x_2^3 + x_3^2$$

Es example, for polynomial of the form

$$5x^5 + 6y^2 + 3z^2 = 0 \tag{1}$$

we have the following surface and roots in complex plane

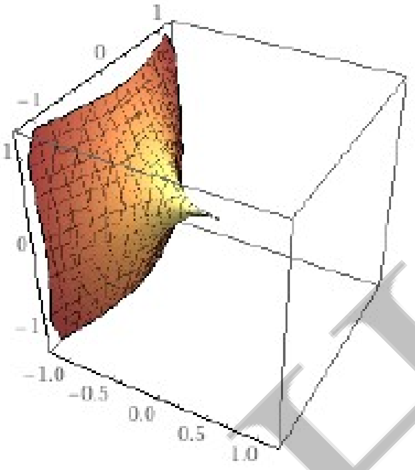

Figure 0.1. Surface of the equation (1).

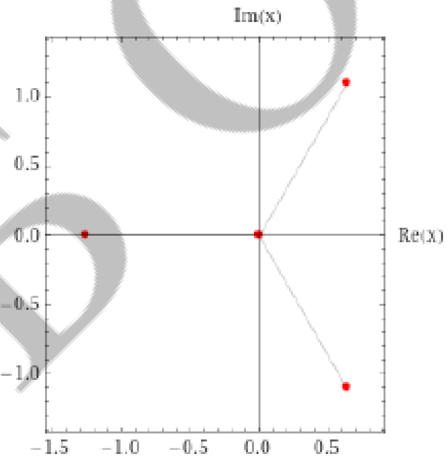

Figure 0.2. Roots of equation (1).

References

[1] Brian R. Greene, C. Vafa, Nicholas P. Warner. Calabi-Yau manifolds and renormalization group flows. *Nucl.Phys.*, B324 1: 371–390, 1989.

[2] C. Vafa, Nicholas P. Warner. Catastrophes and the classification of conformal theories. *Phys. Lett.*, B218 1: 51–58, 1989.

[3] Michihiro Naka, Masatoshi Nozaki. Singular Calabi-Yau Manifolds and ADE Classification of CFTs. *Nucl.Phys.*, B599: 334–360, 2001.

On critical submanifolds of the Willmore energy in four dimensions

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We establish a rigidity result for the critical points, with boundary, of the four dimensional Willmore energy (see [13] where this energy was studied from analytical standpoint). These critical points satisfy a 4-Willmore equation which is a sixth order nonlinear elliptic partial differential equation. We establish several curvature estimates and prove that four dimensional Willmore submanifold with totally geodesic boundary condition are umbilic.

The rigidity of several kinds of submanifolds has been widely studied in literature under different contexts. For instance, while some rigidity results for manifolds with bounded Ricci curvature were obtained in [2] other studies have focused on minimal submanifolds [3, 5, 6, 7, 11, 14, 16], critical points of the Willmore functional [8, 9, 10] and hypersurfaces of constant weighted mean curvature [1, 4, 15]. In [12], McCoy and Wheeler considered surfaces Σ immersed into $\mathbb{R}^3$ which are critical points of the functional

$$\int_\Sigma |\nabla H|^2 \, d\mu$$

and whose second fundamental form satisfies the smallness condition

$$\int_\Sigma |h|^2 \, d\mu \leq \varepsilon$$

where ε is a small universal constant. They obtained the following result.

Theorem 1. *Let $f : \Sigma \to \mathbb{R}^3$ be an immersion satisfying*

$$\Delta^2 H + |h|^2 \Delta H - (h_0)^{ij} \nabla_i H \nabla_j H = 0$$

with the boundary conditions

$$|h| = 0 \quad \text{and} \quad \nabla_\eta H = \nabla_\eta \Delta H = 0.$$

If f also satisfies $\int_\Sigma |h|^2 d\mu \leq \varepsilon$ for some sufficiently small $\varepsilon > 0$, then the immersed surface $f(\Sigma)$ is part of a flat plane, where η is the unit conormal to the boundary of Σ.

Our main result is the following rigidity theorem for critical points of the energy $\mathcal{E}(\Sigma)$.

Theorem 2. *Let $\vec{\Phi} : \Sigma \to \mathbb{R}^m$ be an immersion of a 4-dimensional manifold Σ satisfying $\int_\Sigma |\vec{h}|^2 d\mu \leq \varepsilon$ and $\int_\Sigma |\vec{h}|^4 d\mu \leq \varepsilon$ for some sufficiently small $\varepsilon > 0$. If $\vec{\Phi}$ also satisfies*

$$\vec{\mathcal{W}} = \vec{0} \tag{1}$$

together with the boundary conditions

$$\pi_{\vec{\eta}} \nabla \Delta_\perp \vec{H} = \pi_{\vec{\eta}} \nabla \vec{H} = \vec{0} \quad \text{and} \quad \vec{h} = \vec{0} \tag{2}$$