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## LIST OF TOPICS

- Topological methods in analysis
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- Algebraic methods in geometry
- Differential geometry in the whole
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# On the Carleman function for matrix factorizations of the Helmholtz equation

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This article deals with the construction of the Carleman function for matrix factorizations of the Helmholtz equation in a multidimensional domain.

It is known that the Cauchy problem for elliptic equations is unstable relatively small change in the data, i.e., is incorrect (Hadamard's example). In unstable problems the image of the operator is not closed, therefore the solvability condition can not be written in terms of continuous linear functionals. Thus, in the Cauchy problem for elliptic equations with data on a part of the boundary of the region, the solution is usually unique, the problem is solvable for an everywhere dense set of data, but this set not closed. Consequently, the theory of solvability of such problems is essentially It is more difficult and deeper than the theory of solvability of the Fredholm equations. The first results in this direction appeared only in the mid-1980s in the works of L.A. Aizenberg, A.M. Kytmanov, N.N. Tarkhanov (See, for instance [1]).

Let  $x = (x_1, \dots, x_m), y = (y_1, \dots, y_m)$  be are points of the Euclidean space  $\mathbb{R}^m$  and  $G \subset \mathbb{R}^m$  be a bounded simply-connected domain with piecewise smooth boundary consisting of the plane  $T: y_m = 0$  and of a smooth surface  $S$  lying in the half-space  $y_m > 0$ , that i.e.,  $\partial G = S \cup T$ .

We consider in the domain  $G$  a system of differential equations

$$D \left( \frac{\partial}{\partial x} \right) U(x) = 0, \quad (1)$$

where  $D \left( \frac{\partial}{\partial x} \right)$  is the matrix of first-order differential operators.

We denote by  $A(G)$  the class of vector functions in a domain  $G$  continuous on  $\bar{G} = G \cup \partial G$  and satisfying system (1).

We define the function  $\Phi(y, x; \lambda)$  at  $y \neq x$  by the following equalities:

$$\Phi(y, x; \lambda) = \frac{1}{c_m K(x_m)} \frac{\partial^{k-1}}{\partial s^{k-1}} \int_0^\infty \operatorname{Im} \left[ \frac{K(w)}{w - x_m} \right] \frac{u I_0(\lambda u)}{\sqrt{u^2 + \alpha^2}} du, \quad m = 2k, \quad k \geq 1, \quad (2)$$

$$\Phi(y, x; \lambda) = \frac{1}{c_m K(x_m)} \frac{\partial^{k-1}}{\partial s^{k-1}} \int_0^\infty \operatorname{Im} \left[ \frac{K(w)}{w - x_m} \right] \frac{\cos(\lambda u)}{\sqrt{u^2 + \alpha^2}} du, \quad m = 2k + 1, \quad k \geq 1, \quad (3)$$

where

at  $m = 2k, k \geq 1; c_2 = 2\pi, c_m = (-1)^k 2^{-k} (m-2) \pi \omega_m (k-1)!, I_0(\lambda u) = J_0(i\lambda u)$ —is the Bessel function of the first kind of zero order;

at  $m = 2k + 1, k \geq 1; c_m = (-1)^k 2^{-k} (2k-1)! (m-2) \pi \omega_m, \omega_m$ — the area of a unit sphere in space  $\mathbb{R}^m$ .

In the future, using formulas (2) and (3), we will construct the Carleman matrix for matrix factorizations of the Helmholtz equation in multidimensional bounded domain and based on it we will find an approximate solution to the Cauchy problem in explicit form, using the methodology of previous works (See, for instance [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17] and [18]).

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<b>I. Gelbukh</b> <i>Realization of a graph as the Reeb graph of a Morse, Morse–Bott or round function</i>	<b>38</b>
<b>N. Glazunov</b> <i>Foliations in moduli spaces of abelian varieties and bounded global <math>G</math>-shtukas</i>	<b>40</b>
<b>A. Glebova</b> <i>Geometry of curves in three-dimensional space and invariants of nonlinear differential equations of the second order</i>	<b>41</b>
<b>P. Petrenko, A. Andreev</b> <i>On Orthosymmetric <math>n</math>-morphisms</i>	<b>42</b>
<b>M. Golasinski</b> <i>On homotopy nilpotency of Moore space</i>	<b>43</b>
<b>E. Afanas'eva, A. Golberg</b> <i>Metric viewpoint in mapping theory between Riemannian manifolds</i>	<b>45</b>
<b>S. V. Gryshchuk</b> <i>Monogenic functions with values in commutative complex algebras of the second rank with unity and generalized biharmonic equation with non-zero simple characteristics</i>	<b>46</b>
<b>A. Guale, J. Vielma</b> <i>The Collatz conjecture from an algebraic point of view</i>	<b>47</b>
<b>O. Gutik, P. Khylynskyi</b> <i>On the monoid of cofinite partial isometries of positive integers with a bounded finite noise</i>	<b>48</b>
<b>O. Gutik, M. Mykhalenych</b> <i>On some generalization of the bicyclic monoid</i>	<b>50</b>
<b>Y. Gutman</b> <i>Variational principles for metric mean dimension</i>	<b>51</b>
<b>O. Ye. Hentosh, A. K. Prykarpatski</b> <i>A generalized Lie-algebraic approach to constructing of integrable fractional dynamical systems</i>	<b>52</b>
<b>L. Yanovich, M. Ignatenko</b> <i>On the solution of separate differential equations with variational derivatives of the first and second orders</i>	<b>54</b>
<b>Sh. Sh. Ismoilov</b> <i>Special mean and total curvature of a dual surface in isotropic spaces</i>	<b>57</b>
<b>D. Jumaev, A. Zaitov</b> <i>A topological transformation group of a hyperspace</i>	<b>59</b>
<b>D. A. Juraev</b> <i>On the Carleman function for matrix factorizations of the Helmholtz equation</i>	<b>62</b>
<b>A. Kachurovskii</b> <i>Measuring the rate of convergence in the Birkhoff ergodic theorem</i>	<b>64</b>
<b>O. Kadubov's'kyi</b> <i>Enumeration of topologically non-equivalent functions with one degenerate saddle critical point on triple torus</i>	<b>65</b>
<b>O. Karupu</b> <i>On some properties of moduli of smoothness of conformal mapping of simply connected domains</i>	<b>68</b>
<b>D. Katz</b> <i>Integration over non-rectifiable curves: spirals of high torsion</i>	<b>70</b>
<b>A. A. Katz</b> <i>On real <math>\Sigma^*</math>-algebras of operators</i>	<b>72</b>
<b>L. Fardigola, K. Khalina</b> <i>On controllability problems for the heat equation with variable coefficients controlled by the Dirichlet boundary condition on a half-axis</i>	<b>73</b>