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«Algebraic and geometric methods  
of analysis»

Book of abstracts



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## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
- History and methodology of teaching in mathematics

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НТБ ОНАФТ

## Constructive description of $G$ -monogenic mappings in the algebra of complex quaternions

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Let  $\mathbb{H}(\mathbb{C})$  be the quaternion algebra over the field of complex numbers  $\mathbb{C}$ , whose basis consists of the unit 1 of the algebra and of the elements  $I, J, K$  satisfying the multiplication rules:

$$I^2 = J^2 = K^2 = -1,$$

$$IJ = -JI = K, \quad JK = -KJ = I, \quad KI = -IK = J.$$

In the algebra  $\mathbb{H}(\mathbb{C})$  there exists another basis  $\{e_1, e_2, e_3, e_4\}$  such that multiplication table in a new basis can be represented as

$\cdot$	$e_1$	$e_2$	$e_3$	$e_4$
$e_1$	$e_1$	0	$e_3$	0
$e_2$	0	$e_2$	0	$e_4$
$e_3$	0	$e_3$	0	$e_1$
$e_4$	$e_4$	0	$e_2$	0

The unit of the algebra can be decomposed as  $1 = e_1 + e_2$ .

Let us consider the vectors

$$i_1 = e_1 + e_2, \quad i_2 = a_1 e_1 + a_2 e_2, \quad i_3 = b_1 e_1 + b_2 e_2, \quad (1)$$

where  $a_k, b_k \in \mathbb{C}$ ,  $k = 1, 2$ , which are linearly independent over the field of real numbers  $\mathbb{R}$ . It means that the equality  $\alpha_1 i_1 + \alpha_2 i_2 + \alpha_3 i_3 = 0$  for  $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$  holds if and only if  $\alpha_1 = \alpha_2 = \alpha_3 = 0$ .

In the algebra  $\mathbb{H}(\mathbb{C})$  we consider the linear span

$$E_3 := \{\zeta = xi_1 + yi_2 + zi_3 : x, y, z \in \mathbb{R}\}$$

generated by the vectors  $i_1, i_2, i_3$  over the field  $\mathbb{R}$ .

In the paper [1] we introduced a new class of quaternionic mappings, so-called,  $G$ -monogenic mappings.

A continuous mapping  $\Phi : \Omega_\zeta \rightarrow \mathbb{H}(\mathbb{C})$  (or  $\widehat{\Phi} : \Omega_\zeta \rightarrow \mathbb{H}(\mathbb{C})$ ) is *right- $G$ -monogenic* (or *left- $G$ -monogenic*) in a domain  $\Omega_\zeta \subset E_3$ , if  $\Phi$  (or  $\widehat{\Phi}$ ) is differentiable in the sense of the Gâteaux at every point of  $\Omega_\zeta$ , i. e. for every  $\zeta \in \Omega_\zeta$  there exists an element  $\Phi'(\zeta) \in \mathbb{H}(\mathbb{C})$  (or  $\widehat{\Phi}'(\zeta) \in \mathbb{H}(\mathbb{C})$ ) such that

$$\lim_{\varepsilon \rightarrow 0+0} \left( \Phi(\zeta + \varepsilon h) - \Phi(\zeta) \right) \varepsilon^{-1} = h \Phi'(\zeta) \quad \forall h \in E_3$$

$$\left( \text{or } \lim_{\varepsilon \rightarrow 0+0} \left( \widehat{\Phi}(\zeta + \varepsilon h) - \widehat{\Phi}(\zeta) \right) \varepsilon^{-1} = \widehat{\Phi}'(\zeta) h \quad \forall h \in E_3 \right).$$

We introduce linear functionals  $f_1 : \mathbb{H}(\mathbb{C}) \rightarrow \mathbb{C}$  and  $f_2 : \mathbb{H}(\mathbb{C}) \rightarrow \mathbb{C}$  by setting

$$f_1(e_1) = f_1(e_3) = 1, \quad f_1(e_2) = f_1(e_4) = 0,$$

$$f_2(e_2) = f_2(e_4) = 1, \quad f_2(e_1) = f_2(e_3) = 0.$$

Denote by  $f_k(E_3) := \{f_k(\zeta) : \zeta \in E_3\}$  for  $k = 1, 2$ .

Note that the points  $(x, y, z) \in \mathbb{R}^3$  corresponding to the non-invertible elements  $\zeta = xi_1 + yi_2 + zi_3 \in E_3$  lie on the straight lines

$$L^1 : x + y \operatorname{Re} a_1 + z \operatorname{Re} b_1 = 0, \quad y \operatorname{Im} a_1 + z \operatorname{Im} b_1 = 0,$$

$$L^2 : x + y \operatorname{Re} a_2 + z \operatorname{Re} b_2 = 0, \quad y \operatorname{Im} a_2 + z \operatorname{Im} b_2 = 0$$

in the three-dimensional space  $\mathbb{R}^3$ .

Denote by

$$D_1 := f_1(\Omega_\zeta) \subset \mathbb{C}, \quad D_2 := f_2(\Omega_\zeta) \subset \mathbb{C}.$$

In the following theorems we established constructive description of all  $G$ -monogenic mappings using four analytic functions of the complex variable.

**Theorem 1.** *Let a domain  $\Omega$  be convex in the direction of the straight lines  $L^1$  and  $L^2$  and let  $f_1(E_3) = f_2(E_3) = \mathbb{C}$ . Then every right- $G$ -monogenic mapping  $\Phi : \Omega_\zeta \rightarrow \mathbb{H}(\mathbb{C})$  has the form*

$$\Phi(\zeta) = F_1(\xi_1)e_1 + F_2(\xi_2)e_2 + F_3(\xi_1)e_3 + F_4(\xi_2)e_4,$$

where  $F_1$  and  $F_3$  are functions of the variable  $\xi_1 := x + ya_1 + zb_1$  analytic in the domain  $D_1$ , and  $F_2$  and  $F_4$  are functions of the variable  $\xi_2 := x + ya_2 + zb_2$  analytic in the domain  $D_2$ .

**Theorem 2.** *Let a domain  $\Omega$  be convex in the direction of the straight lines  $L^1$  and  $L^2$  and let  $f_1(E_3) = f_2(E_3) = \mathbb{C}$ . Then every left- $G$ -monogenic mapping  $\hat{\Phi} : \Omega_\zeta \rightarrow \mathbb{H}(\mathbb{C})$  has the form*

$$\hat{\Phi}(\zeta) = \hat{F}_1(\xi_1)e_1 + \hat{F}_2(\xi_2)e_2 + \hat{F}_3(\xi_2)e_3 + \hat{F}_4(\xi_1)e_4,$$

where  $\hat{F}_1$  and  $\hat{F}_4$  are functions of the variable  $\xi_1 := x + ya_1 + zb_1$  analytic in the domain  $D_1$ , and  $\hat{F}_2$  and  $\hat{F}_3$  are functions of the variable  $\xi_2 := x + ya_2 + zb_2$  analytic in the domain  $D_2$ .

#### REFERENCES

- [1] V. S. Shpakivskyi, T. S. Kuzmenko. On one class of quaternionic mappings. *Ukr. Math. J.*, 68(1) : 127 – 143, 2016.

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