International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Investigation of the connection between different models of topologies on a finite set

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One of the unsolved problems of discrete mathematics is the problem of counting all topologies on a finite set. Topologies on a finite set were modeled using various mathematical objects (graphs, partial order relations, Boolean functions and their normal forms, $(0,1)$-matrices of a special form, etc.). In [1] topologies were studied using the topology vector, the concept of which was introduced in [2]. In [3], in addition to the topology vector, Boolean functions and a maximal 2-CNF were used. The question arises about the relationship between the objects of different models, which can be used both to continue research and to verify the results. This issue was partially raised by us in our work [5].

In this paper, we consider the connection between models in the form of $(0,1)$-matrices of a special form and in the form of ordered sets $(M_1, M_2, ..., M_n)$ of minimal neighborhoods of elements of a given ordered finite set $X = (x_1, x_2, ..., x_n)$ (using such sets, one can instantly pass to topology vectors - ordered sets of integers α_k, which were effectively used in [1,2]).

According to [4] the $(0,1)$-matrix σ_{ij}, where $1 \leq i, j \leq n$, corresponds to some topology on X (in this case, this matrix is called a grid, and its order is the order of the grid) if and only if the following conditions are true:

1) $\sigma_{ij} = 1$, if $x_i \in \overline{x}_j$,
2) $\sigma_{ij} = 0$ in the other case

(Here the symbol $\overline{x}_j$ indicates the closure of a point x_j in a given topology).

Let, in the i-line of the matrix $\sigma_{ir_1} = 1$,$\sigma_{ir_2} = 1$,...,$\sigma_{ir_k} = 1$, the other elements be equal to zero. From $\sigma_{ir_1} = 1$ follows $x_i \in \overline{x}_{r_1}$, and then $M_i \cap \{x_{r_1}\} \neq \varnothing$. So, $x_{r_1} \in M_i$. Similarly, with $\sigma_{ir_2} = 1$ we get $x_{r_2} \in M_i$ etc. Hence, $M_i \supseteq \{x_i, x_{r_1}, ..., x_{r_k}\}$. On the other hand, for an element x_p from M_i the inclusion $x_i \in \overline{x}_p$ is obvious. Then $\sigma_{ip} = 1$. Thus, the i-line of the $(0,1)$-matrix corresponds to the minimal neighborhood of the element x_i.

The connection found made it possible to prove some properties of networks using minimal neighborhoods. In particular, the following properties of networks:

1) $\sigma_{ii} = 1$ at all $i = 1, ..., n$;
2) if $\sigma_{ir} = 1$ and $\sigma_{rj} = 1$, then $\sigma_{ij} = 1$;
3) The network σ defines T_0-topology on X (is T_0-network) if and only if $\sigma_{ij}\sigma_{ji} = 0$ at $i \neq j$.

Using these and other properties of networks and their connection with sets of minimal neighborhoods (bases of topologies), we enumerate all possible networks of T_0-topologies on a 4-element set and find the total number of T_0-topologies and the number 355 all of the topologies on this set using the well-known formula from work [6].

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Normal subgroups of iterated wreath products of symmetric groups and alternating with symmetric groups

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In this research we continue our previous investigation of wreath product normal structure [1].

The lattice of normal subgroups and their properties for finite iterated wreath products $S_{n_1} \wr \ldots \wr S_{n_m}$, $n, m \in \mathbb{N}$ are found. Special classes of normal subgroups and their orders and generators are found. Further, the monolith of these wreath products is investigated by us.

Let $k(\pi)$ be the number of cycles in decomposition of permutation π of degree n.

The number $n - k(\pi)$ is denoted by $dec(\pi)$, and is called a decrement [2] of permutation π.

As well known [2] the minimal number of transpositions in factorization of a permutation π on transpositions is happen to be equal to $dec(\pi)$. We set $dec(e) = 0$. Therefore the decrement of n-cycle is $n-1$.

If $\pi_1, \pi_2 \in S_n$, then the following formula holds:

$$dec(\pi_1 \cdot \pi_2) = dec(\pi_1) + dec(\pi_2) - 2m, m \in \mathbf{N}, \tag{1}$$

where m is number of joint simplifying transpositions in π_1 and π_2.

The trivial subgroup of S_n we denote by E.

Definition 1. The set of elements from $S_n \wr S_n, n \geqslant 5$ or $n = 3$ of the tableaux form: $[e]_1, [a_1, a_2, \ldots, a_n]_2$, satisfying the following condition

$$\sum_{i=1}^{n} dec([a_i]_2) = 2k, k \in \mathbb{N}, \tag{2}$$

we will call set of type $\widetilde{A}^{(2)}$ and denote this set by $E \wr \widetilde{A}_n$. For brevity of notation this subgroup be also denoted by $\widetilde{A}_n^{(2)}$. It follows directly from the definition that the set of these elements