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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

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ІНТЕРНАЦІОНАЛЬНИЙ
ЦЕНТР СПІВРОБОТИ

Asymptotically best possible Lebesgue inequalities on the classes of generalized Poisson integrals

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Denote by $C_{\beta}^{\alpha,r}C$, $\alpha > 0$, $r > 0$, (see, e.g., [1]) the set of all 2π -periodic functions, such that for all $x \in \mathbb{R}$ can be represented in the form of convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} P_{\alpha,r,\beta}(x-t)\varphi(t)dt, \quad a_0 \in \mathbb{R}, \quad \varphi \perp 1, \quad (1)$$

where $\varphi \in C$, and $P_{\alpha,r,\beta}(t)$ is a generalized Poisson kernel of the form

$$P_{\alpha,r,\beta}(t) = \sum_{k=1}^{\infty} e^{-\alpha k^r} \cos\left(kt - \frac{\beta\pi}{2}\right), \quad \alpha > 0, \quad r > 0, \quad \beta \in \mathbb{R}.$$

If f and φ are connected with a help of equality (1), then the function f in this equality is called the generalized Poisson integral of the function φ and is denoted by $J_{\beta}^{\alpha,r}(\varphi)$. The function φ in the equality (1) is called the generalized derivative of the function f and is denoted by $f_{\beta}^{\alpha,r}$.

By $\rho_n(f; x)$ we denote the deviation of the function f from its partial Fourier sum of order $n - 1$:

$$\rho_n(f; x) := f(x) - S_{n-1}(f; x),$$

where

$$S_{n-1}(f; x) = \frac{a_0}{2} + \sum_{k=1}^{n-1} (a_k \cos kx + b_k \sin kx),$$

$$a_k = a_k(f) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos ktdt, \quad b_k = b_k(f) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin ktdt,$$

and by $E_n(f)_C$ we denote the best uniform approximation of the function f by elements of the subspace τ_{2n-1} of trigonometric polynomials $t_{n-1}(\cdot)$ of the order $n - 1$:

$$E_n(f)_C := \inf_{t_{n-1} \in \tau_{2n-1}} \|f - S_{n-1}(f)\|_C.$$

The norms $\|\rho_n(f; \cdot)\|_C$ can be estimated via $E_n(f)_C$, using the Lebesgue inequality

$$\|\rho_n(f; \cdot)\|_C \leq \left(\frac{4}{\pi^2} \ln n + \mathcal{O}(1) \right) E_n(f)_C, \quad n \in \mathbb{N}. \quad (2)$$

On the whole space C the inequality (2) is asymptotically exact. At the same time for the sets of functions $C_{\beta}^{\alpha,r}C$ the inequality (2) is not asymptotically exact.

We establish the asymptotically best possible Lebesgue-type inequalities for the functions $f \in C_{\beta}^{\alpha,r}C$, in which for all n , starting from the number $n_1 = n_1(\alpha, r)$, an additional term is estimated by absolute constant.

For arbitrary $\alpha > 0$, $r \in (0, 1)$ we denote by $n_1 = n_1(\alpha, r)$ the smallest integer $n \in \mathbb{N}$, such that

$$\frac{1}{\alpha r} \frac{1}{n^r} \left(1 + \ln \frac{\pi n^{1-r}}{\alpha r} \right) + \frac{\alpha r}{n^{1-r}} \leq \frac{1}{(3\pi)^3}. \quad (3)$$

Theorem 1. *Let $\alpha > 0$, $r \in (0, 1)$, $\beta \in \mathbb{R}$ and $n \in \mathbb{N}$. Then, for any function $f \in C_{\beta}^{\alpha, r} C$ and all $n \geq n_1(\alpha, r)$ the following inequality holds*

$$\|\rho_n(f; \cdot)\|_C \leq e^{-\alpha n^r} \left(\frac{4}{\pi^2} \ln \frac{n^{1-r}}{\alpha r} + \gamma_n \right) E_n(f_{\beta}^{\alpha, r})_C. \quad (4)$$

Moreover, for arbitrary function $f \in C_{\beta}^{\alpha, r} C$ one can find a function $F(x) = F(f, n, x)$ from the set $C_{\beta}^{\alpha, r} C$, such that $E_n(F_{\beta}^{\alpha, r})_C = E_n(f_{\beta}^{\alpha, r})_C$, such that for $n \geq n_1(\alpha, r)$ the equality holds

$$\|\rho_n(F; \cdot)\|_C = e^{-\alpha n^r} \left(\frac{4}{\pi^2} \ln \frac{n^{1-r}}{\alpha r} + \gamma_n \right) E_n(f_{\beta}^{\alpha, r})_C. \quad (5)$$

In (4) and (5) for the quantity $\gamma_n = \gamma_n(\alpha, r, \beta)$ the estimate holds $|\gamma_n| \leq 20\pi^4$.

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