

International
Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
(17.03.1928-18.12.2019)

May 25-28, 2021
Odesa, Ukraine

LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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A topological transformation group of a hyperspace

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Let X be a compact Hausdorff space. By $\exp X$ we denote a set of all nonempty closed subsets of X . A family of sets of the view

$$O\langle U_1, \dots, U_n \rangle = \{F \in \exp X : F \subset \bigcup_{i=1}^n U_i, F \cap U_1 \neq \emptyset, \dots, F \cap U_n \neq \emptyset\}$$

forms a base of a topology on $\exp X$, where U_1, \dots, U_n are open nonempty sets in X . This topology is called *the Vietoris topology*. A space $\exp X$ equipped with Vietoris topology is called *hyperspace* of X . For a compact space X its hyperspace $\exp X$ is also a compact space.

Let $f: X \rightarrow Y$ be continuous map of compacts, $F \in \exp X$. We put

$$(\exp f)(F) = f(F).$$

This equality defines a map $\exp f: \exp X \rightarrow \exp Y$. For a continuous map f the map $\exp f$ is continuous. Really, it follows from the formula [2]

$$(\exp f)^{-1}O\langle U_1, \dots, U_m \rangle = O\langle f^{-1}(U_1), \dots, f^{-1}(U_m) \rangle$$

what one can check directly. Note that if $f: X \rightarrow Y$ is an epimorphism, then $\exp f$ is also an epimorphism.

For a Tychonoff space X we put

$$\exp_\beta X = \{F \in \exp \beta X : F \subset X\}.$$

It is clear, that $\exp_\beta X \subset \exp \beta X$. Consider the set $\exp_\beta X$ as a subspace of the space $\exp \beta X$. For a Tychonoff spaces X the space $\exp_\beta X$ is also a Tychonoff space with respect to the induced topology.

For a continuous map $f: X \rightarrow Y$ of Tychonoff spaces we put

$$\exp_\beta f = (\exp \beta f)|_{\exp_\beta X},$$

where $\beta f: \beta X \rightarrow \beta Y$ is the Stone-Cěch compactification of f (it is unique).

For a Tychonoff space X put

$$\exp(\text{Homeo}(X)) = \{\exp(g) : g \in \text{Homeo}(X)\}.$$

Proposition 1. *For an arbitrary Tychonoff space X we have*

$$\exp(\text{Homeo}(X)) \subset \text{Homeo}(\exp(X)).$$

Note that the inclusion cannot be reversed.

Example 1. Let $X = \{a, b\}$ be a two-point discrete space. Then $\exp X$ is three point discrete space. There exist only two homeomorphisms of X onto itself: $h, h': X \rightarrow X$, defined by the rules $h(a) = a, h(b) = b$ and $h'(a) = b, h'(b) = a$. At the same time $\exp X$ has six different homeomorphisms four of them could not be generated by h and h' .

For a topological transformation group (G, X, α) we put

$$\exp(G) = \{\exp(\alpha_g) : g \in G\},$$

here $\alpha_g(x) = g(x)$.

Let U_g be an open in G neighbourhood of an element $g \in G$. we define a set $U_{\exp(\alpha_g)} = \{\exp(\alpha_h) : h \in U_g\}$ and put

$$\mathfrak{B}_{\exp(\alpha_g)} = \{U_{\exp(\alpha_g)} : U_g \in \tau_G\},$$

here τ_G is the topology on the space G . It is easy to check that the family $\mathfrak{B}_{\exp(\alpha_g)}$ forms a neighbourhood system of the point $\exp(\alpha_g) \in \exp(G)$.

Theorem 1. *The set $\exp(G)$ is a topological group with respect to the operation $\exp(\alpha_{g_1})\exp(\alpha_{g_2}) = \exp(\alpha_{g_1g_2})$. Moreover, $\exp(\alpha_e)$ is a unit of the group $\exp(G)$ and $\exp(\alpha_g)^{-1} = \exp(\alpha_{g^{-1}})$, $g \in G$.*

Now for α it is possible to define the action $\alpha^{\exp} : \exp(G) \times \exp(X) \rightarrow \exp(X)$ by the rule

$$\alpha^{\exp}(\exp(\alpha_g), F) = \exp(\alpha_g)(F).$$

Proposition 2. *For the topological transformation groups (G, X, α) , the triple $(\exp(G), \exp(X), \alpha^{\exp})$ is a topological transformation groups.*

Proposition 3. *If the set $A \subset X$ is G -invariant, then the set $\exp(A)$ is $\exp(G)$ -invariant.*

Proposition 4. *For a topological transformation group (G, X, α) , we have*

$$\ker \alpha^{\exp} = \exp(\ker \alpha).$$

Here $\ker \alpha^{\exp} = \{\exp(\alpha_g) \in \exp(G) : \exp(\alpha_g)(F) = F, \forall F \in \exp(X)\}$, $\exp(\ker \alpha) = \{\exp(\alpha_g) \in \exp(G) : g \in \ker \alpha\}$.

Proposition 4 immediately implies

Corollary 5. *The action α^{\exp} is effective if and only if the action α is effective.*

Note that for the transitive action α of the group G on the space X , the action α^{\exp} induced from α may not be transitive.

Example 6. Let $X = \{x_1, x_2, x_3\}$ be the discrete topological space (all three points are different). Let

$$G = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \right\}$$

– the discrete topological group of permutations of the index set $\{1, 2, 3\}$. The action $\alpha : G \times X \rightarrow X$ of the group G on the space X is defined by the rule $\alpha(g, x_i) = x_{g(i)}$. Then α is a transitive action. Moreover, $\alpha_g(x_i) = x_{g(i)}$. It is clear that $\exp(\alpha_g)(\{x_1, x_2, x_3\}) = \{x_1, x_2, x_3\}$ for each $g \in G$. Thus, for no closed subset $F \subset X$ there is no element $\exp(\alpha_g)$ of the group $\exp(G)$ for which $\exp(\alpha_g)(F) = \Phi$, here $\Phi = \{x_1, x_2, x_3\}$, $F \neq \Phi$. Therefore, the action α^{\exp} is not transitive.

Example 6 shows that the action of the group $\exp(G)$ on the space $\exp(X)$ may not be free, although the action of the group G on the space X is free. But, nevertheless, the following is true.

Proposition 7. *Let $X = \{x_1, \dots, x_n\}$ be a finite discrete space, G an arbitrary permutation group (supplied by the discrete topology) of the set X . Then, for each free action α of the group G on the space X , the corresponding action α^{\exp} of the group $\exp(G)$ on the space $\exp(X)$ is semi-free. In this case, the only point in the space $\exp(X)$ that remains motionless under the action of all elements of $\exp(G)$ is the set $\{x_1, \dots, x_n\}$.*

It is clear that if G is a compact group, then $\exp(G)$ is also a compact group.

Theorem 8. *The action $\alpha^{\exp}: \exp(G) \times \exp(X) \rightarrow \exp(X)$ of the compact group $\exp(G)$ on the space $\exp(X)$ is a closed map.*

The next statement follows from Theorem 8.

Corollary 9. *If G is a compact group and X is some G -space, then for any closed set $A \subset \exp(X)$ the set $\exp(G)(A)$ is closed in $\exp(X)$ and for compact A the set $\exp(G)(A)$ is compact.*

Theorem 10. *If $f: X \rightarrow Y$ is an equivariant map of one G -space to another, then $\exp(f): \exp(X) \rightarrow \exp(Y)$ is also an equivariant map of $\exp(G)$ -spaces.*

The normality of the functor \exp and Theorem 10 imply

Corollary 11. *If $f: X \rightarrow Y$ is an equivalence of G -spaces X and Y , then $\exp(f): \exp X \rightarrow \exp Y$ is an equivalence of $\exp(G)$ -spaces $\exp X$ and $\exp Y$.*

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I. Gelbukh <i>Realization of a graph as the Reeb graph of a Morse, Morse–Bott or round function</i>	38
N. Glazunov <i>Foliations in moduli spaces of abelian varieties and bounded global G-shtukas</i>	40
A. Glebova <i>Geometry of curves in three-dimensional space and invariants of nonlinear differential equations of the second order</i>	41
P. Petrenko, A. Andreev <i>On Orthosymmetric n-morphisms</i>	42
M. Golasinski <i>On homotopy nilpotency of Moore space</i>	43
E. Afanas'eva, A. Golberg <i>Metric viewpoint in mapping theory between Riemannian manifolds</i>	45
S. V. Gryshchuk <i>Monogenic functions with values in commutative complex algebras of the second rank with unity and generalized biharmonic equation with non-zero simple characteristics</i>	46
A. Guale, J. Vielma <i>The Collatz conjecture from an algebraic point of view</i>	47
O. Gutik, P. Khylynskyi <i>On the monoid of cofinite partial isometries of positive integers with a bounded finite noise</i>	48
O. Gutik, M. Mykhalenych <i>On some generalization of the bicyclic monoid</i>	50
Y. Gutman <i>Variational principles for metric mean dimension</i>	51
O. Ye. Hentosh, A. K. Prykarpatski <i>A generalized Lie-algebraic approach to constructing of integrable fractional dynamical systems</i>	52
L. Yanovich, M. Ignatenko <i>On the solution of separate differential equations with variational derivatives of the first and second orders</i>	54
Sh. Sh. Ismoilov <i>Special mean and total curvature of a dual surface in isotropic spaces</i>	57
D. Jumaev, A. Zaitov <i>A topological transformation group of a hyperspace</i>	59
D. A. Juraev <i>On the Carleman function for matrix factorizations of the Helmholtz equation</i>	62
A. Kachurovskii <i>Measuring the rate of convergence in the Birkhoff ergodic theorem</i>	64
O. Kadubovs'kyi <i>Enumeration of topologically non-equivalent functions with one degenerate saddle critical point on triple torus</i>	65
O. Karupu <i>On some properties of moduli of smoothness of conformal mapping of simply connected domains</i>	68
D. Katz <i>Integration over non-rectifiable curves: spirals of high torsion</i>	70
A. A. Katz <i>On real Σ^*-algebras of operators</i>	72
L. Fardigola, K. Khalina <i>On controllability problems for the heat equation with variable coefficients controlled by the Dirichlet boundary condition on a half-axis</i>	73