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and Geometric  
Methods of Analysis**

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## LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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# Realization of groups as fundamental groups of orbits of smooth maps

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Let  $M$  be a connected compact oriented surface and  $P$  be a real line  $\mathbb{R}$  or a circle  $S^1$ . Note, that the group  $\mathcal{D}(M)$  of diffeomorphisms of  $M$  naturally acts on the space of smooth maps  $C^\infty(M, P)$  by the rule  $(f, h) \mapsto f \circ h$ , where  $h \in \mathcal{D}(M)$ ,  $f \in C^\infty(M, P)$ . For  $f \in C^\infty(M, P)$  denote by  $\mathcal{O}(f)$  the orbit of  $f$  under this action. Let  $\mathcal{M}(M, P)$  be the set of isomorphism classes of fundamental groups  $\pi_1 \mathcal{O}(f)$  of orbits of Morse maps  $f: M \rightarrow P$ .

S. Maksymenko [1, 2] and B. Feshchenko [3] introduced the sets of isomorphism classes  $\mathcal{B}$  and  $\mathcal{T}$  of groups generated by direct products and certain wreath products. They have proved that  $\mathcal{M}(M, P) \subset \mathcal{B}$  if  $M$  is different from a 2-sphere  $S^2$  and a 2-torus  $T^2$ , and  $\mathcal{M}(T^2, \mathbb{R}) \subset \mathcal{T}$ . We proved that these inclusions are equalities.

**Definition 1.** Let  $\mathcal{B}$  be a minimal class of groups satisfying the following conditions:

- 1)  $1 \in \mathcal{B}$ ;
- 2) if  $G_1, G_2 \in \mathcal{B}$ , then  $G_1 \times G_2 \in \mathcal{B}$ ;
- 3) if  $G \in \mathcal{B}$  and  $n \geq 1$ , then  $G \wr_n \mathbb{Z} \in \mathcal{B}$ .

Let also  $\mathcal{T}$  be the set of isomorphism classes of groups consisting of groups of the form  $G \wr_{n,m} \mathbb{Z}^2$ , where  $G \in \mathcal{B}$  and  $n, m \geq 1$ .

Let also  $\mathcal{B}^0$  be a subclass of  $\mathcal{B}$  consisting of groups  $(A \times B) \wr_n \mathbb{Z}$ , where  $A, B \in \mathcal{B} \setminus \{1\}$  and  $n \geq 1$ . Note, that  $\mathcal{B}^0 \subset \mathcal{B} \subset \mathcal{T}$ .

Denote by  $\mathcal{F}(M, P)$  the space of smooth maps  $f \in C^\infty(M, P)$  satisfying the following two conditions:

- (1) all critical points of  $f$  belong to the interior of  $M$ , and  $f$  takes constant values on each connected component of the boundary of  $M$ ;
- (2) for each critical point  $z$  of  $f$  its germ at  $z$  is smoothly equivalent to some non-zero homogeneous polynomial  $\mathbb{R}^2 \rightarrow \mathbb{R}$  of degree  $\geq 2$  without multiple factors.

The set of all Morse maps from  $M$  to  $P$  is denoted by  $Morse(M, P)$ . For each map  $f \in \mathcal{F}(M, P)$  we can define the (continuous) function  $\varepsilon_f$  from the set of connected components of the boundary  $\partial M$  to  $\{\pm 1\}$ , which takes the value  $-1$  on the boundary component if  $f$  has a local minimum on this component, and  $+1$  if  $f$  has a local maximum on this component. Let  $\mathcal{E}_M$  be the set of all continuous functions  $\varepsilon: \partial M \rightarrow \{\pm 1\}$ . For  $\varepsilon \in \mathcal{E}_M$  we denote by  $\mathcal{F}(M, P, \varepsilon)$  ( $Morse(M, P, \varepsilon)$ ) subset of  $\mathcal{F}(M, P)$  ( $Morse(M, P)$ ) of functions  $f$ , for which  $\varepsilon_f = \varepsilon$ .

Denote

$$\mathcal{G}_X(M, P, \varepsilon) := \{\pi_1 \mathcal{O}(f, X) \mid f \in \mathcal{F}(M, P, \varepsilon)\},$$

$$\mathcal{M}_X(M, P, \varepsilon) := \{\pi_1 \mathcal{O}(f, X) \mid f \in \text{Morse}(M, P, \varepsilon)\},$$

$$\mathcal{G}^\Psi := \{\pi_1 \mathcal{O}(f) \mid f \in \mathcal{F}(T^2, \mathbb{R}), \text{ the Kronrod-Reeb graph } \Gamma_f \text{ is a tree}\},$$

$$\mathcal{M}^\Psi := \{\pi_1 \mathcal{O}(f) \mid f \in \text{Morse}(T^2, \mathbb{R}), \text{ the Kronrod-Reeb graph } \Gamma_f \text{ is a tree}\},$$

$$\mathcal{G}^O := \{\pi_1 \mathcal{O}(f) \mid f \in \mathcal{F}(T^2, \mathbb{R}), \text{ the Kronrod-Reeb graph } \Gamma_f \text{ has an unique cycle}\},$$

$$\mathcal{M}^O := \{\pi_1 \mathcal{O}(f) \mid f \in \text{Morse}(T^2, \mathbb{R}), \text{ the Kronrod-Reeb graph } \Gamma_f \text{ has an unique cycle}\}.$$

**Theorem 2.** (1) *Let  $M$  be a connected compact oriented surface distinct from 2-torus and 2-sphere, and let  $\varepsilon: \partial M \rightarrow \{\pm 1\}$  be an arbitrary map from  $\mathcal{E}_M$ . Then*

a) *if  $M = S^1 \times [0, 1]$ , and  $\varepsilon$  is constant, i.e takes the same value on components of the boundary  $\partial M$ , then  $\mathcal{M}_{\partial M}(M, P, \varepsilon) = \mathcal{G}_{\partial M}(M, P, \varepsilon) = \mathcal{B} \setminus \{1\}$ ,*

b) *if  $M = S^1 \times [0, 1]$  and  $\varepsilon$  takes different values on the components of the boundary  $\partial M$  or  $M \neq S^1 \times [0, 1]$ , then  $\mathcal{M}_{\partial M}(M, P, \varepsilon) = \mathcal{G}_{\partial M}(M, P, \varepsilon) = \mathcal{B}$ .*

(2) *There are equalities  $\mathcal{M}^\Psi = \mathcal{G}^\Psi = \mathcal{T}$ ,  $\mathcal{M}^O = \mathcal{G}^O = \mathcal{B}^O$ .*

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<b>Kh. F. Kholturayev</b> <i>Perfect metrizable of the functor of idempotent measures</i>	<b>75</b>
<b>Y. Khomych</b> <i>Quasiareal deformation of surfaces of positive Gauss curvature</i>	<b>77</b>
<b>V. Kiosak, O. Lesechko</b> <i>Canonical infinitesimal deformations of metrics of pseudo-Riemannian spaces</i>	<b>78</b>
<b>R. Salimov, B. Klishchuk</b> <i>On the behavior at infinity of ring <math>Q</math>-homeomorphisms</i>	<b>79</b>
<b>T. Kolomiets, A. Pogorui</b> <i>Elements of probability theory and measures with values in hypercomplex algebras</i>	<b>81</b>
<b>N. Konovenko</b> <i>The invariants of planar 3-webs with respect to group of symplectic diffeomorphisms, for the case of the conformal group</i>	<b>84</b>
<b>E. Kudryavtseva</b> <i>Topology of spaces of smooth functions and gradient-like flows with prescribed singularities on surfaces</i>	<b>85</b>
<b>G. Kuduk</b> <i>Nonlocal problem with integral conditions for homogeneous system of partial differential equations of second order</i>	<b>87</b>
<b>I. Kuznietsova, Yu. Soroka</b> <i>Realization of groups as fundamental groups of orbits of smooth maps</i>	<b>88</b>
<b>K. Gürlebeck, D. Legatiuk</b> <i>Modified quaternionic operator calculus and its application to micropolar elasticity</i>	<b>90</b>
<b>S. Maksymenko, E. Polulyakh</b> <i>On non-Hausdorff manifolds of dimension 1</i>	<b>92</b>
<b>S. Maksymenko</b> <i>Symplectomorphisms preserving smooth functions on surfaces</i>	<b>93</b>
<b>M. Maloid-Hliebova</b> <i>Second classical Zariski topology of multiplicative module</i>	<b>94</b>
<b>I. Marko</b> <i>Incomplete spaces of idempotent measures</i>	<b>95</b>
<b>N. Mazurenko, M. Zarichnyi</b> <i>Hyperspaces of convex sets related to idempotent mathematics</i>	<b>96</b>
<b>A. Mednykh</b> <i>Volumes of knots and links in spaces of constant curvature</i>	<b>98</b>
<b>R. Mohseni, R. A. Wolak</b> <i>Twistor spaces on foliated manifolds</i>	<b>99</b>
<b>P. Mormul</b> <i>Two problems in nonholonomic geometry (in quest of a co-worker)</i>	<b>100</b>
<b>F. Mukhamadiev</b> <i>The local <math>\tau</math>-density of a linearly ordered spaces</i>	<b>101</b>
<b>T. Obikhod</b> <i>Entropy and phase transitions in Calabi-Yau space</i>	<b>102</b>
<b>A. Orevkova</b> <i>Reducing singularities of smooth functions to normal forms</i>	<b>104</b>
<b>T. Osipchuk</b> <i>On <math>m</math>-convexity and <math>m</math>-semiconvexity of sets in Euclidean spaces</i>	<b>106</b>
<b>V. Ostrovskiy, O. Ostrovska, D. Proskurin, Yu. Samoilenko</b> <i>On representations of <math>q_{ij}</math>-commuting isometries</i>	<b>108</b>
<b>J.F. Peters</b> <i>Homotopic Nerve Complexes with Free Group Presentations</i>	<b>110</b>
<b>P. Laurain, M. Petrace</b> <i>Uniform measures in Euclidean space</i>	<b>112</b>