



International  
Scientific Conference



# Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of  
**Dvytro Grave**  
(25.08.1863 - 19.12.1939)  
Academician of the Ukrainian  
Academy of Sciences, the  
first director of the Institute of  
Mathematics of NAS of Ukraine

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Odesa, Ukraine

## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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The support of any  $A \in \bar{I}(X)$  is the set

$$\text{supp}(A) = \text{Cl} (\{ x \in X \mid \exists t > 0, (x, t) \in A \}).$$

The set  $\in \bar{I}(X)$  is endowed with the Hausdorff metric induced by the max-metric on  $X \times [0, 1]$ . Some hyperspaces of countable closed sets in metric spaces are considered in [3]. Denote by  $A'$  is the derived set (the set of all accumulation points) of  $A$ .

We introduce the following spaces of idempotent measures:

$$\bar{A}_n(X) = \{A \in \bar{I}(X) \mid 1 \leq |(\text{supp}(A))'| \leq n\} \quad (n \in \mathbb{N});$$

$$\bar{A}_\omega(X) = \bigcup_{n \in \mathbb{N}} \bar{A}_n(X).$$

By  $\mathcal{K}(X \times [0, 1])$  we denote the hyperspace of all countable compact subsets of  $X \times [0, 1]$ .

**Theorem 2.** *Let  $X$  be a separable metric space. Then:*

- (1) *For  $n \in \mathbb{N}$ , the space  $\bar{A}_n(X)$  is  $F_{\sigma\delta}(\mathcal{K}(X \times [0, 1]))$ ;*
- (2)  *$\bar{A}_\omega(X)$  is  $F_{\sigma\delta\sigma}(\mathcal{K}(X \times [0, 1]))$ .*

The proof of this statement is based on some results from [3].

We then apply some characterization results of infinite-dimensional topology (see [4]) to describe the topology of spaces  $\bar{I}(X)$  for noncompact locally compact separable metric spaces  $X$ .

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## SKT hyperbolic and Gauduchon hyperbolic compact complex manifolds

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**Definition 1.** Let  $X$  be a compact complex manifold with  $\dim_{\mathbb{C}} X = n$ , and  $\omega$  be a metric on  $X$ : be a  $C^\infty$  positive definite  $(1, 1)$ -form on  $X$ .

- i)  $\omega$  is *Kähler*, if  $d\omega = 0$ .
- ii)  $\omega$  is *balanced*, if  $d\omega^{n-1} = 0$ .
- iii)  $\omega$  is *Gauduchon*, if  $\bar{\partial}\partial\omega^{n-1} = 0$ , such a metric always exists on a compact complex manifold.

iv)  $\omega$  is **SKT** (or pluriclosed), if  $\partial\bar{\partial}\omega = 0$

Let  $\pi_X : \tilde{X} \rightarrow X$  be the universal cover of  $X$  and  $\tilde{\omega} = \pi_X^*\omega$  be the Hermitian metric on  $\tilde{X}$  that is the lift of  $\omega$ . Recall that a  $C^\infty$   $k$ -form  $\alpha$  on  $X$  is said to be  $\tilde{d}$ (bounded) with respect to  $\omega$  if  $\pi_X^*\alpha = d\beta$  on  $\tilde{X}$  for some  $C^\infty$   $(k-1)$ -form  $\beta$  on  $\tilde{X}$  that is bounded w.r.t.  $\tilde{\omega}$ . (See [1] and [2]). In general, we propose the following definition which generalizes that of  $\tilde{d}$ -bounded of a differential form.

**Definition 2.** a  $C^\infty$   $k$ -form  $\phi$  on  $X$  is said to be  $(\widetilde{\partial + \bar{\partial}})$ -bounded with respect to  $\omega$  if  $\pi_X^*\phi = \partial\alpha + \bar{\partial}\beta$  on  $\tilde{X}$  for some  $C^\infty$   $(k-1)$ -forms  $\alpha$  and  $\beta$  on  $\tilde{X}$  that are bounded w.r.t.  $\tilde{\omega}$ .

M. Gromov introduced in one of his seminal papers [1] the notion of *Kähler hyperbolicity* for a compact Kähler manifold  $X$ . The manifold  $X$  is called *Kähler hyperbolic* if  $X$  admits a Kähler metric  $\omega$  whose lift  $\tilde{\omega}$  to the universal cover  $\tilde{X}$  of  $X$  can be expressed as

$$\tilde{\omega} = d\alpha$$

for a *bounded* 1-form  $\alpha$  on  $\tilde{X}$ . As pointed out by Gromov, it is not hard to see that the Kähler hyperbolicity implies the Kobayashi hyperbolicity. The Kähler hyperbolicity is generalized in [2] to what we call **balanced hyperbolicity**. This is done by replacing the Kähler metric in the Kähler hyperbolicity by a *balanced metric*. Meanwhile, a compact complex  $n$ -dimensional manifold  $X$  is said to be balanced hyperbolic if it carries a balanced metric  $\omega$  such that  $\omega^{n-1}$  is  $\tilde{d}$ -bounded. The Brody hyperbolicity is replaced by what we call **divisorial hyperbolicity**. A compact complex manifold  $X$  is called *divisorially hyperbolic* if there exists no non-trivial holomorphic map from  $\mathbb{C}^{n-1}$  to  $X$  satisfying certain *subexponential volume growth condition*.

We introduce the following

**Definition 3.** Let  $X$  be a compact complex manifold with  $\dim_{\mathbb{C}}X = n$ . A Hermitian metric  $\omega$  on  $X$  is said to be

- (1) **SKT hyperbolic** if  $\omega$  is SKT and  $(\widetilde{\partial + \bar{\partial}})$ -bounded with respect to  $\omega$ . The manifold  $X$  is said to be SKT hyperbolic if it carries a *SKT hyperbolic metric*.
- (2) **Gauduchon hyperbolic** if  $\omega^{n-1}$  is  $(\widetilde{\partial + \bar{\partial}})$ -bounded with respect to  $\omega$ . The manifold  $X$  is said to be Gauduchon hyperbolic if it carries a *Gauduchon hyperbolic metric*.

**Lemma 4.** *The following implication holds:*

$$X \text{ is Kähler hyperbolic} \implies X \text{ is SKT hyperbolic}$$

$$\Downarrow$$

$$X \text{ is balanced hyperbolic} \implies X \text{ is Gauduchon hyperbolic}$$

The following results are taken from [3]

**Theorem 5.** *Every SKT hyperbolic compact complex manifold is Kobayashi hyperbolic.*

**Remark 6.** An immediate observation is that, since a *SKT hyperbolic* manifold  $X$  contains no rational curves, then by Mori's cone theorem we get  $K_X$  is nef.

**Theorem 7.** *Every Gauduchon hyperbolic compact complex manifold is divisorially hyperbolic.*

**Theorem 8.** *Let  $X$  be a compact complex **SKT hyperbolic** manifold with  $\dim_{\mathbb{C}} X = n$ . Let  $\pi : \tilde{X} \rightarrow X$  be the universal cover of  $X$  and  $\tilde{\omega} := \pi^* \omega$  the lift to  $\tilde{X}$  of a *SKT hyperbolic* metric  $\omega$  on  $X$ . Fix a primitive  $L^2_{\tilde{\omega}}$ -form  $\phi$  on  $\tilde{X}$  of bidegree  $(p, q)$  with  $p + q = n - 1$  such that*

$$\partial\phi = 0, \quad \bar{\partial}\phi = 0.$$

*Then  $\phi = 0$ .*

**Corollary 9.** *Let  $\phi$  be a  $(n-1, 0)$ -form (respectively a  $(0, n-1)$ -form) on a connected complete manifold  $(\tilde{X}, \tilde{\omega})$  such that*

$$\phi \in L^2(\tilde{X}), \quad \partial\phi = 0, \quad \bar{\partial}\phi = 0.$$

*If  $\tilde{\omega} = \partial\alpha + \bar{\partial}\beta$  where  $\alpha$  and  $\beta$  are bounded 1-forms on  $\tilde{X}$ , then*

$$\phi = 0.$$

**Theorem 10.** *Let  $(X, \omega)$  be a complete Kähler manifold of dimension  $2n$  and  $\omega = \partial\alpha + \bar{\partial}\beta$  where  $\alpha$  and  $\beta$  are respectively a bounded  $(0, 1)$  and  $(1, 0)$  forms on  $X$ . Then every  $L_2$ -form  $\Psi$  on  $X$  of degree  $p \neq m$  satisfies the inequality*

$$\langle \psi, \Delta\psi \rangle \geq \lambda_0^2 \langle \psi, \psi \rangle,$$

*where  $\lambda_0$  is a strictly positive constant which depends only on  $n = \dim X$ ,  $\alpha$  and  $\beta$ .*

**Corollary 11.** *Let  $(\tilde{X}, \tilde{\omega})$  be a connected complete Kähler manifold. If  $\tilde{\omega} = \partial\alpha + \bar{\partial}\beta$  where  $\alpha$  and  $\beta$  are bounded 1-forms on  $\tilde{X}$ , then  $\mathcal{H}_{\Delta_{\tilde{\omega}}}^p(\tilde{X}, \mathbb{C}) = 0$ , unless  $p = n$ .*

This is a new conjecture.

**Conjecture 12.** *If a compact complex manifold admits a balanced hyperbolic metric and an *SKT hyperbolic* metric, then it admit a Kähler hyperbolic metric.*

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