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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
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НТБ ОНАФТ

Construction and topological properties of the closed extension topology

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Let (X, τ) be a topological space and let X^* be a superset of X . Then the family

$$\tau^* = \{V \subset X^* \mid V \subset X^* \setminus X \text{ or } V = (X^* \setminus X) \cup U, U \in \tau\}$$

is a topology for X^* , which is called *the closed extension topology of X to X^** . Indeed, if all of the sets $U_\alpha \in \tau^*$, $\alpha \in T$, lies in $X^* \setminus X$, then their union is also contained in $X^* \setminus X$. Otherwise, this union has the form $(X^* \setminus X) \cup U$ for some $U \in \tau$. If the index set T is finite and at least one of U_α lies in $X^* \setminus X$, then the intersection $\bigcap_{\alpha \in T} U_\alpha$ is also contained in $X^* \setminus X$. Otherwise, this intersection has the form $(X^* \setminus X) \cup U$ for some $U \in \tau$.

Thus the open sets of X^* are all subsets of $X^* \setminus X$ and all unions $(X^* \setminus X) \cup U$ where U is open in X . Respectively, closed sets in X^* are all supersets of X and all closed sets in X .

Example 1. Let X be a topological space and let A be a proper subset of X . Then A -excluded topology for X is the closed extension topology of the indiscrete space A to X .

Proposition 2. *The closed extension topology τ^* of a topological space (X, τ) to X^* is supremum of topology $\sigma = \{\emptyset\} \cup \{(X^* \setminus X) \cup V, V \in \tau\}$ and X -excluded topology for X^* .*

A map $f : X \rightarrow Y$ of topological spaces (X, τ) and (Y, σ) is called *inducing*, if the topology τ is induced by σ and f .

Theorem 3. *Let (X, τ) be a topological space, let X^* be a superset of the set X and let τ^* be the closed extension topology of X to X^* . Then the natural embedding $X \ni x \xrightarrow{i} x \in X^*$ is closed inducing map. In particular, (X, τ) is closed subspace of (X^*, τ^*) .*

Unlike extension topology [2] closed extension topology (like open extension topology [3]) is not transitive. Hence the natural embedding $X \ni x \xrightarrow{i} x \in X^*$ is not quotient, i. e. the closed extension topology is not quotient topology with respect to τ and i .

Example 4. Consider nested sets $X = \{a\}$, $X^* = \{a, b\}$, $X^{**} = \{a, b, c\}$. Then for the discrete topology τ for X we have $(\tau^*)^* = \{\emptyset, X^{**}, \{c\}, \{b, c\}\} \neq \tau^{**} = \{\emptyset, X^{**}, \{b\}, \{c\}, \{b, c\}\}$.

Let us describe a base of the least cardinality of the closed extension topology and local one with respect to it.

Proposition 5. *A base of the least cardinality of the closed extension topology of a space X to X^* has the form $\beta^* = \{\{x\}, (X^* \setminus X) \cup U \mid x \in X^* \setminus X, U \in \beta\}$ where β is a base of the least cardinality of the space X .*

Proposition 6. *Let τ^* be the closed extension topology of a space X to X^* . Then a local base of the least cardinality at any point $x \in X^*$ has the form $\{\{x\}\}$ for $x \in X^* \setminus X$, and $\{(X^* \setminus X) \cup U \mid U \in \beta_x\}$ where β_x is a local base of the least cardinality at point x in X , for $x \in X$.*

The following Proposition gives an explicit description of the interior, the closure, the sets of isolated and limit points of an arbitrary set of a topological space with the closed extension topology, and necessary and sufficient conditions for density and nowhere density of a given set.

Proposition 7. *Let τ^* be the closed extension topology of a space X to X^* and let $A \subset X^*$. Then:*

1) *the interior of A in X^* is equal to $\text{Int}_{X^*} A = (X^* \setminus X) \cup \text{Int}_X(A \cap X)$, where $\text{Int}_X(A \cap X)$ is the interior of the intersection $A \cap X$ in X , when $A \supset X^* \setminus X$, and $\text{Int}_{X^*} A = A \setminus X$ otherwise;*

- 2) the closure of A in X^* has the form $\overline{A}_{X^*} = \overline{A}_X$, where \overline{A}_X is the closure of A in X , when $A \subset X$, and $\overline{A}_{X^*} = A \cup X$ otherwise;
- 3) the set of isolated points of A in X^* is calculated by formula $I_{X^*}(A) = I_X(A)$, where $I_X(A)$ is the set of isolated points of A in X , when $A \subset X$, and $I_{X^*}(A) = A \setminus X$ otherwise;
- 4) the set of limit points of A in X^* is equal to A'_X , where A'_X is the set of limit points of A in X , when $A \subset X$, and $A'_{X^*} = X$ otherwise;
- 5) if $X^* \setminus X \neq \emptyset$, then A is dense in X^* if and only if $A \supset X^* \setminus X$; otherwise A is dense in X^* if and only if A is dense in X .
- 6) A is nowhere dense in X^* if and only if $A \subset X$.

Now we proceed with the study of topological properties of the closed extension topology.

Theorem 8. Let τ^* be the closed extension topology of a space X to X^* and $X^* \neq X$. Then X^* is path connected and thus connected.

Corollary 9. Every topological space can be embedded as closed subspace in a path connected (and thus connected) topological space.

The next fact follows directly from propositions 5 and 6.

Theorem 10. Let τ^* be the closed extension topology of a space X to X^* . The topological space X^* is first countable if and only if the space X is first countable. The space X^* is second countable if and only if the space X is second countable and the complement $X^* \setminus X$ is at most countable.

Theorem 11. Let τ^* be the closed extension topology of a space X to X^* . The topological space X^* is Lindelöf (compact) if and only if the space X is Lindelöf (compact).

Theorem 12. Let τ^* be the closed extension topology of a space X to X^* and $X^* \neq X$. The topological space X^* is separable if and only if the complement $X^* \setminus X$ is at most countable. The set $X^* \setminus X$ is at most countable dense set in X^* of the least cardinality.

Theorem 13. Let τ^* be the closed extension topology of a space X to X^* and $X^* \neq X$. The topological space X^* is T_0 if and only if the space X is T_0 . The space X^* is T_4 if and only if every two nonempty closed sets in X intersect (a stronger condition than T_4 separation axiom). The space X^* is not T_i for $i = 1, 2, 3$. In particular, X^* is not regular, normal and metrizable.

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