International
Scientific Conference

# Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

## List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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(2) a set of critical points $\Sigma_f$ of $f$ is a disjoint union of smooth submanifolds of $M$ and $\Sigma_f \subset \mathrm{Int}(M)$,

(3) for each connected component $C$ of $\Sigma_f$ and each critical point $p \in C$ there exist a local chart $(U, \phi : U \to \mathbb{R}^2)$ near $p$ and a chart $(V, \psi : V \to \mathbb{R})$ near $f(p) \in P$ such that $f(U) \subset V$ and a local representation $\psi \circ f \circ \phi^{-1} : \phi(U) \to \psi(V)$ of $f$ is
   (a) either a homogeneous polynomial $f_p : \mathbb{R}^2 \to \mathbb{R}$ of degree $\deg f_p \geq 2$ having no multiple factors,
   (b) or is given by $f_C(x, y) = \pm y^{n_C}$ for some $n_C \in \mathbb{N}_{\geq 2}$ depending of $C$.

Note that the class $\mathcal{F}(M, P)$ contains the class of $P$-valued Morse-Bott functions on $M$.

**Theorem 2** (Theorem 1.2 [1]). *For a function $f \in \mathcal{F}(M, P)$ the group $\mathcal{S}_{\mathrm{id}}(f)$ is contractible if $f$ has at least one saddle or $M$ is non-oriented, otherwise $\mathcal{S}_{\mathrm{id}}(f)$ is homotopy equivalent to $S^1$.*

## References

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# On direct limits of Minkowski's balls, domains, and their critical lattices

**Nikolaj Glazunov**
(Glushkov Institute of Cybernetics NASU, Kiev,
Institute of Mathematics and Informatics Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria.)
*E-mail:* glanm@yahoo.com

We construct direct systems of Minkowski, Davis and Chebyshev-Cohn balls and domains, direct systems of their critical lattices and calculate their direct limits. By (general) Minkowski balls we mean (two-dimensional) balls in $\mathbb{R}^2$ of the form

$$D_p : \ |x|^p + |y|^p \leq 1, \ p \geq 1. \tag{1}$$

From the proof of Minkowski's conjecture [1, 2, 3, 4, 5, 8] in notations [8, 9] we have next expressions for critical determinants and their lattices:

**Theorem 1.** (1) $\Delta(D_p) = \Delta_p^{(0)} = \Delta(p, \sigma_p) = \frac{1}{2}\sigma_p, \ 2 \leq p \leq p_0;$
(2) $\sigma_p = (2^p - 1)^{1/p}$,
(3) $\Delta(D_p) = \Delta_p^{(1)} = \Delta(p, 1) = 4^{-\frac{1}{p}} \frac{1+\tau_p}{1-\tau_p}, \ 1 \leq p \leq 2, \ p \geq p_0,$
(4) $2(1 - \tau_p)^p = 1 + \tau_p^p, \ 0 \leq \tau_p < 1,$
*where $p_0$ is a real number that is defined unique by conditions $\Delta(p_0, \sigma_p) = \Delta(p_0, 1), \ 2,57 < p_0 < 2,58, \ p_0 \approx 2.5725$*
*For their critical lattices respectively $\Lambda_p^{(0)}$, $\Lambda_p^{(1)}$ next conditions satisfy: $\Lambda_p^{(0)}$ and $\Lambda_p^{(1)}$ are two $D_p$-admissible lattices each of which contains three pairs of points on the boundary of $D_p$ with the property that $(1, 0) \in \Lambda_p^{(0)}$, $(-2^{-1/p}, 2^{-1/p}) \in \Lambda_p^{(1)}$,*

Denote by $V(D_p)$ the volume (area) of $D_p$.

**Proposition 2.** *The volume of Minkowski ball $D_p$ is equal $4\frac{(\Gamma(1+\frac{1}{p}))^2}{\Gamma(1+\frac{2}{p})}$.*

**Proof.** (by Minkowski). Let $x^P + y^p \leq 1, x \geq 0, y \geq 0$. Put $x^p = \xi, y^p = \eta$.

$$V(D_p) = \frac{4}{p^2} \iint \xi^{\frac{1}{p}-1} \eta^{\frac{1}{p}-1} d\xi d\eta, \tag{2}$$

where the integral extends to the area

$$\xi + \eta \leq 1,\ \xi \geq 0,\ \eta \geq 0.$$

Expression (2) can be represented in terms of Gamma functions, and we get

$$V(D_p) = 4\frac{(\Gamma(1+\frac{1}{p}))^2}{\Gamma(1+\frac{2}{p})}.$$

□

We consider balls of the form

$$D_p:\ |x|^p + |y|^p \leq 1,\ p \geq 1,$$

and call such balls with $1 < p < 2$ *Minkowski balls.* Continuing this, we consider the following classes of balls and circles.

- *Davis balls*: $|x|^p + |y|^p \leq 1$ for $p_0 > p \geq 2$;
- *Chebyshev-Cohn balls*: $|x|^p + |y|^p \leq 1$ for $p \geq p_0$;

Let $D$ be a fixed bounded symmetric about origin convex body (*centrally symmetric convex body* for short) with volume $V(D)$.

**Proposition 3.** [6]. *If $D$ is symmetric about the origin and convex, then $2D$ is convex and symmetric about the origin.*

**Corollary 4.** *Let $m$ be integer $m \geq 0$ and $n$ be natural greater $m$. If $2^m D$ centrally symmetric convex body then $2^n D$ is again centrally symmetrc convex body.*

**Proof.** Induction.
We consider the following classes of balls (see above) and domains.

- *Minkowski domains*: $2^m D_p$, integer $m \geq 1$, for $1 \leq p < 2$;
- *Davis domains*: $2^m D_p$, integer $m \geq 1$, for $p_0 > p \geq 2$;
- *Chebyshev-Cohn domains*: $2^m D_p$, integer $m \geq 1$, for $p \geq p_0$;

**Proposition 5.** *Let $m$ be integer, $m \geq 1$. If $\Lambda$ is the critical lattice of the ball $D_p$ than the sublattice $\Lambda_{2^m}$ of index $2^m$ is the critical lattice of the domain $2^{m-1} D_p$.*

The direct system of Minkowski balls and domains has the form (3), where the multiplication by 2 is the continuous mapping

$$D_p \xrightarrow{2} 2D_p \xrightarrow{2} 2^2 D_p \xrightarrow{2} \cdots \xrightarrow{2} 2^m D_p \xrightarrow{2} \cdots \tag{3}$$

The direct system of critical lattices has the form (4), where the multiplication by 2 is the homomorphism of abelian groups

$$\Lambda_p \xrightarrow{2} 2\Lambda_p \xrightarrow{2} 2^2 \Lambda_p \xrightarrow{2} \cdots \xrightarrow{2} 2^m \Lambda_p \xrightarrow{2} \cdots \tag{4}$$

In our considerations we have direct systems of Minkowski balls, Minkowski domains and direct systems of critical lattices with respective maps and homomorphisms. Let $\mathbb{Q}_2$ and $\mathbb{Z}_2$ be respectively the field of 2-adic numbers and its ring of integers. Denote the corresponding direct limits by $D_p^{dirlim}$ and by $\Lambda_p^{dirlim}$.

**Proposition 6.** $D_p^{dirlim} = \varinjlim 2^m D_p \in (\mathbb{Q}_2/\mathbb{Z}_2)D_p = (\bigcup_m \frac{1}{2^m}\mathbb{Z}_2/\mathbb{Z}_2)D_p$.

**Proposition 7.** $\Lambda_p^{dirlim} = \varinjlim 2^m \Lambda_p \in (\mathbb{Q}_2/\mathbb{Z}_2)\Lambda_p = (\bigcup_m \frac{1}{2^m}\mathbb{Z}_2/\mathbb{Z}_2)\Lambda_p$.

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# On KB(Kantorovich-Banach) spaces and KB operators


**Omer Gok**

(Yildiz Technical University, Faculty of Arts and Sciences, Mathematics Department, Esenler, Istanbul, TURKEY)

*E-mail:* gok@yildiz.edu.tr


Let $E$ be a Banach lattice and $X$ be a Banach space. $E$ is said to be a $KB$ space if a positive increasing sequence in the closed unit ball of $E$ converges. Every $KB$-space has order continuous norm, but the converse is not true in general. $c_0$ has order continuous norm, but $c_0$ is not a $KB$-space. For $1 \leq p < \infty$, $L^p$-spaces are $KB$-spaces.

An operator $T : E \to X$ is said to be a $KB$ operator if for every positive increasing sequence $(x_n)$ in the closed unit ball of $E$ , the sequence $(Tx_n)$ converges. An operator $T : X \to X$ is called demicompact if, for every bounded sequence $(x_n)$ in $X$ such that $(x_n - Tx_n)$ converges to $x \in X$ , there is a convergent subsequence of $(x_n)$. An operator $T : X \to X$ is said to be a demi Dunford-Pettis if, for every sequence $(x_n)$ in $X$ such that $(x_n)$ converges to zero weakly and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$. Every Dunford-Pettis operator is demi Dunford-Pettis operator. An operator $T : E \to E$ is called a demi $KB$ operator if, for every positive increasing sequence $(x_n)$ in the closed unit ball of $E$ such that $(x_n - Tx_n)$ is norm convergent to $x \in E$, there is a norm convergent subsequence of $(x_n)$. For the identity operator $I : E \to E$, the operator $2I$ is a demi $KB$-operator. Every $KB$ operator is a demi $KB$ operator.