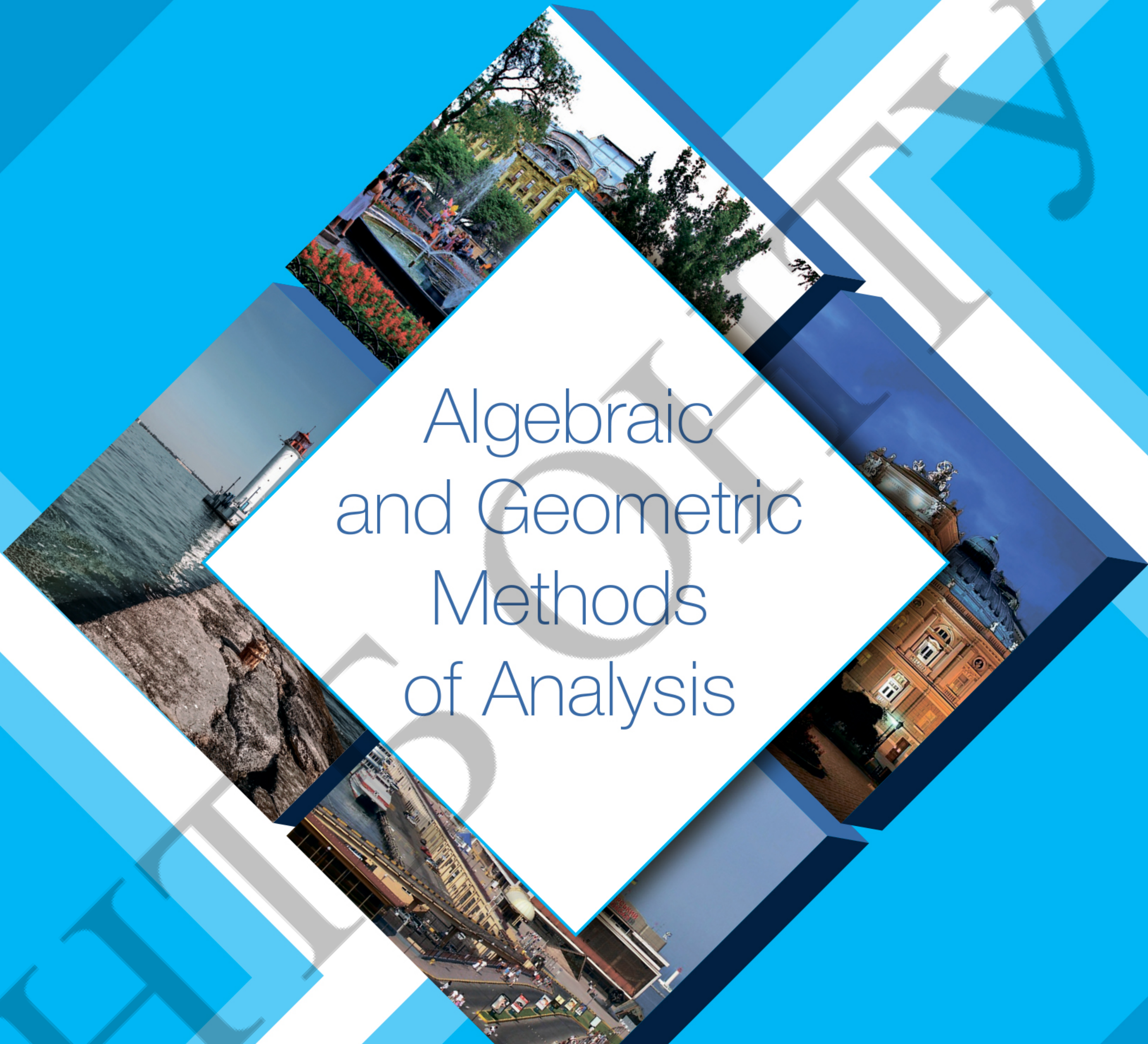
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Vladimir Balan** (*Bucharest, Romania*)
- **Taras Banakh** (*Lviv, Ukraine*)
- **Dmytro Bolotov** (*Kharkiv, Ukraine*)
- **Vyacheslav Boyko** (*Kyiv, Ukraine*)
- **Yulia Fedchenko** (*Odesa, Ukraine*)
- **Oleg Gutik** (*Lviv, Ukraine*)
- **Olena Karlova** (*Chernivtsi, Ukraine*)
- **Volodymyr Kiosak** (*Odesa, Ukraine*)
- **Nadiia Konovenko** (*Odesa, Ukraine*)
- **Volodymyr Lyubashenko** (*Kyiv, Ukraine*)
- **Sergiy Maksymenko** (*Kyiv, Ukraine*)
- **Koji Matsumoto** (*Yamagata, Japan*)
- **Piotr Mormul** (*Warsaw, Poland*)
- **Maryna Nesterenko** (*Kyiv, Ukraine*)
- **Roman Popovych** (*Kyiv, Ukraine*)
- **Alexandr Prishlyak** (*Kyiv, Ukraine*)
- **Aleksandr Savchenko** (*Kherson, Ukraine*)

Organizing commitee

- **Nadiia Konovenko** (*Odesa, Ukraine*)
- **Yuliya Fedchenko** (*Odesa, Ukraine*)
- **Mykola Lysynskiy** (*Kyiv, Ukraine*)
- **Bohdan Mazhar** (*Kyiv, Ukraine*)
- **Sergiy Maksymenko** (*Kyiv, Ukraine*)
- **Alexandr Prishlyak** (*Kyiv, Ukraine*)

References

[1] A. I. Stepanets, *Methods of Approximation Theory* VSP: Leiden, Boston 2005.

[2] O. I. Stepanets', A.S. Serdyuk, A.L. Shydlich, *On some new criteria for infinite differentiability of periodic functions*, Ukr. Math. J., 59, No. 10, 1399-1409 (2007).

Fundamental solution of non-Archimedean pseudo-differential equation of p-adic argument

Mariia Serdiuk

(Institute of Mathematics of the National Academy of Sciences of Ukraine)

E-mail: mariia.v.serdiuk@gmail.com

The Vladimirov-Taibleson pseudo-differential operator D^α plays a role of a differential operator in the p-adic analysis (see [1, 3]). The analogue of the wave equation for radial functions in t on non-Archimedean spaces

$$D_t^\alpha u - D_x^{\alpha,n} u = 0 \tag{1}$$

was studied in [2].

In present work the fundamental solution of a more general Cauchy problem for the functions of two p-adic variables, radial in t, was found. The main result is stated in Theorem 2. Theorem 3 proves the uniqueness of the solution in the Lizorkin space of locally constant functions $\Phi(\mathbb{Q}_p^n)$, and Theorem 4 gives an estimate of the norm of the solution in L^1-space.

Let $0 < \alpha < 1$, $\beta > 0$. Consider the eigenvalue problem

$$D^\alpha u = \lambda u, \ \lambda = p^{\beta N}, \ N \in \mathbb{Z}, \tag{2}$$

where u is not identically zero.

We also suppose that

$$\beta = K\alpha \text{ for some } K \in \mathbb{N}. \tag{3}$$

Proposition 1. *If the condition (3) holds, the equation (2) has the set of solutions in $\Phi(\mathbb{Q}_p)$ of the following form for $N \in \mathbb{Z}$:*

$$u_N(t) = \begin{cases} C_N p^{KN}(1 - \frac{1}{p}), & |t|_p \le p^{-KN}; \\ -C_N p^{KN-1}, & |t|_p = p^{-KN+1}; \\ 0, & |t|_p \ge p^{-KN+2}. \end{cases} \tag{4}$$

Let $0 < \alpha < 1$, $\beta > 0$. We consider the Cauchy problem

$$D_{|t|_p}^\alpha u(|t|_p, x) - D_x^\beta u(|t|_p, x) = 0, \ (t, x) \in \mathbb{Q}_p^+ \times \mathbb{Q}_p^n, \tag{5}$$

$$u(0, x) = u_0(x), \ x \in \mathbb{Q}_p^n, \tag{6}$$

where $u = u(|t|_p, x)$ is a radial function with respect to t, $n \ge 1$.

Theorem 2. *Let $0 < \alpha < 1$, $\beta > 0$ such that the condition (3) holds. Suppose that the function u_0 is in $\Phi(\mathbb{Q}_p)^n$. Then the Cauchy problem (5)-(6) has a solution $u = u(|t|_p, x)$, radial in t, that belongs to the space $\Phi(\mathbb{Q}_p^+)$ for each $x \in \mathbb{Q}_p$, and belongs to $\Phi(\mathbb{Q}_p^n)$ for each $t \in \mathbb{Q}_p^+$.*

If the condition (3) does not hold, then the equation (5) has only a zero solution $u(t, x) \equiv 0$, $t, x \in \mathbb{Q}_p^+ \times \mathbb{Q}_p$.

The solution built in the proof of Theorem 2 has the following form

$$u(|t|_p, x) = (\mathcal{F}^{-1}_{\xi\to x}\hat{u})(|t|_p, x) = \left((\mathcal{F}^{-1}_{\xi\to x}b) * u_0\right)(|t|_p, x),\ (t, x) \in \mathbb{Q}_p^+ \times \mathbb{Q}_p^n, \tag{7}$$

We consider the problem (5)-(6) in the class of generalized functions, radial in t.

Denote by $\Phi'(\mathbb{Q}_p^+, \Phi'(\mathbb{Q}_p^n))$ the set of distributions over the test function space $\Phi(\mathbb{Q}_p^n)$, with values in $\Phi'(\mathbb{Q}_p^n)$.

Theorem 3. *Let $F \in \Phi'(\mathbb{Q}_p^+, \Phi'(\mathbb{Q}_p^n))$ be a generalised solution of the equation (5), that is*

$$\langle\langle F, D_t^\alpha\varphi_1\rangle, \varphi_2\rangle = \langle\langle F, \varphi_1\rangle, D_x^\beta\varphi_2\rangle,$$

for any $\varphi_1 \in \Phi(\mathbb{Q}_p^+)$, $\varphi_2 \in \mathbb{Q}_p$. If F is radial in t, then $F \in \mathcal{D}(\mathbb{Q}_p^+, \Phi'(\mathbb{Q}_p^n))$. If, in addition, $F(0, x) = 0$, then $F(t, x) \equiv 0$.

It follows from Theorem 3 that the solutions of the Cauchy problems constructed in Theorem 2 are unique in the class of radial in t, bounded locally constant functions.

Theorem 4. *Suppose that the conditions of Theorem 2 hold. Then the solution of the problem (5)-(6), defined in (7), satisfies the following estimate in $L^1(\mathbb{Q}_p^n)$ in variable x*

$$||u(|t|_p, \cdot)||_{L_1(\mathbb{Q}_p^n)} \le p^{2n\gamma}||u_0||_{L_1(\mathbb{Q}_p^n)}, \tag{8}$$

where $\gamma \ge \dfrac{2}{K}$ is a positive constant.

References

[1] S. Albeverio, A. Yu. Khrennikov, and V. M. Shelkovich, Theory of p-Adic Distributions, Cambridge University Press, 2010.

[2] A.N. Kochubei, A Non-Archimedean Wave Equation, *Pacif. J. Math*, **235(2)** (2007), 245-261.

[3] V. S. Vladimirov, I. V. Volovich and E. I. Zelenov, *p-Adic Analysis and Mathematical Physics*, World Scientific, Singapore, 1994.

On the semigroup of injective monoid endomorphisms of a some extension of the bicyclic semigroup

Marko Serivka

(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)

E-mail: marko.serivka@lnu.edu.ua

Oleg Gutik

(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)

E-mail: oleg.gutik@lnu.edu.ua

In this paper we shall follow the semigroup terminology of [5].

By ω we denote the set of all non-negative integers.

Let $\mathscr{P}(\omega)$ be the family of all subsets of ω. For any $F \in \mathscr{P}(\omega)$ and any integer n we put $n + F = \{n + k\colon k \in F\}$ if $F \neq \varnothing$ and $n + \varnothing = \varnothing$. A subfamily $\mathscr{F} \subseteq \mathscr{P}(\omega)$ is called ω*-closed* if $F_1 \cap (-n + F_2) \in \mathscr{F}$ for all $n \in \omega$ and $F_1, F_2 \in \mathscr{F}$. For any $a \in \omega$ we denote $[a) = \{x \in \omega\colon x \geqslant a\}$.

On the set $\boldsymbol{B}_\omega = \omega \times \omega$ we define the semigroup operation "$\cdot$" in the following way

$$(i_1, j_1) \cdot (i_2, j_2) = \begin{cases} (i_1 - j_1 + i_2, j_2), & \text{if } j_1 \leqslant i_2; \\ (i_1, j_1 - i_2 + j_2), & \text{if } j_1 \geqslant i_2. \end{cases}$$

It is well known that the bicyclic monoid is isomorphic to the semigroup $\boldsymbol{B}_\omega$.