

International
Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
(17.03.1928-18.12.2019)

May 25-28, 2021
Odesa, Ukraine

LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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Enumeration of topologically non-equivalent functions with one degenerate saddle critical point on triple torus

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Let $(N, \partial N)$ be a smooth surface with the edge ∂N (∂N can be empty). Let $C^\infty(N)$ denote the space of infinitely differentiable functions on N with edge $\partial N = \partial_- N \cup \partial_+ N$, all critical points of which are isolated and lie in the interior of N , and, furthermore, on the connected components of the edge $\partial_- N$ ($\partial_+ N$) the functions from $C^\infty(N)$ take the same values a (b accordingly).

Two functions f_1 and f_2 from the space $C^\infty(N)$ are called topologically equivalent if there are homeomorphisms $h : N \rightarrow N$ and $h' : R^1 \rightarrow R^1$ (h' preserves orientation) such that $f_2 = h' \circ f_1 \circ h^{-1}$. If h preserves of the orientation, the functions f_1 and f_2 are called topologically conjugated (eg. [2]) or O -topologically equivalent (eg. [6]).

It is known [2] that a function $f \in C^\infty(N)$, in a certain neighborhood of its isolated critical point $x \in N$ (which is not a local extremum) for which the topological type of level lines changes in passing through x , is reduced by a continuous change of coordinates to the form $f = \operatorname{Re} z^n + c$, $n \geq 2$ (are called «essentially» critical point) or $f = \operatorname{Re} z$ if the topological type of level lines does not change in passing through x . The number of essentially critical points x_i of the function f , together with the values of n_i (exponents in there presentation $f = \operatorname{Re} z^{n_i} + c_i$ in the neighborhoods of the critical points x_i), are called the topological singular type of the function f .

The problem of the topological equivalence of functions from the class $C^\infty(N)$ with the fixed number of critical points was completely solved by V.V. Sharko in [3] and it was established that a finite number of topologically nonequivalent functions of this class exist. However, it should be noted that the task about calculation of topologically non-equivalent functions with the fixed topological singular type is rather complicated and is still **unsolved**.

When considering functions from the class $C(M_g) \subset C^\infty(N)$ that possess **only one** essentially critical point x_0 (degenerate critical point of the saddle type) in addition to local maxima and minima on oriented surface M_g of genus $g \geq 0$, then the problem of counting the number of such non-equivalent functions is greatly simplified. It is well known [2] that $\forall f \in C(M_g)$ the Poincare index of a critical point x_0 , is equal $\operatorname{ind}^f(x_0) = 1 - n$, where $n = 2g - 1 + \lambda$ and λ is a total number of local maxima and minima.

Let $C_n(M_g)$ be a class of functions from $C(M_g)$ which, in addition to local maxima and minima, have only one essentially critical point, whose the Poincare index is equal $(1 - n)$. Denote the class of functions from $C(M_g)$ that possess one essentially critical point, k local maxima and l local minima by $C_{k,l}(M_g)$. Then it is clear that $\forall f \in C_{k,l}(M_g)$ $n = 2g - 1 + k + l$.

In the general case, for natural g, k, l (or k, l , and $n = 2g + k + l - 1$), the problem of calculating the number of topologically non-equivalent functions from the class $C_{k,l}(M_g)$ also has proved to be quite a difficult and unsolved problem to date.

The task of calculating the number of topologically non-equivalent functions from the class $C_{1,1}(M_g)$ (for genus $g \geq 1$) was completely solved in [4]. In [5], for natural k and l solved completely the problems of calculating the numbers O -topologically and topologically non-equivalent functions from the class $C_{k,l}(M_0)$ (on two-dimensional sphere).

In [6], [7] solved completely the problems of calculating the numbers O -topologically and topologically non-equivalent functions from the class $C_{1,n-2}(M_1)$ and $C_{1,n-4}(M_2)$ accordingly.

In general case, for functions from the class $C_{1,n-2g}(M_g)$, the task is also still unsolved.

By using the results of [1] we can establish the validity of the following statement

Theorem 1. For natural $n \geq 7$ the number $d^*(n)$ of O -topologically non-equivalent functions from the class $C_{1,n-6}(M_3)$ can be calculated by the relation

$$d^*(n) = \frac{1}{n} \left(d(n) + \sum_{j|n, j \in \{2;3;4;6;7;8;9;12\}} \phi(j) \cdot \rho \left(n, \frac{n}{j} \right) \right), \quad (1)$$

where: $\phi(q)$ is the Euler totient function;

$$d(n) = \frac{1}{72} C_{n+1}^8 \cdot (9n^4 - 18n^3 - 57n^2 + 34n + 80); \quad (2)$$

$\forall j \in N : \frac{n}{j} \notin N$ the value $\rho \left(n, \frac{n}{j} \right) \equiv 0$; $\forall j \in \{2; 3; 4; 6; 7; 8; 9; 12\} : \frac{n}{j} \in N$ the value of $\rho \left(n, \frac{n}{j} \right)$ can be calculated by the relations

$$\begin{aligned} \rho \left(n, \frac{n}{12} \right) &= \frac{n}{12}, & \rho \left(n, \frac{n}{9} \right) &= \frac{2n}{9}, & \rho \left(n, \frac{n}{8} \right) &= \frac{n}{4}, & \rho \left(n, \frac{n}{7} \right) &= \frac{5n}{7}, & \rho \left(n, \frac{n}{4} \right) &= \frac{n(n-4)(n+40)}{384}, \\ \rho \left(n, \frac{n}{6} \right) &= \frac{n(n-6)}{72}, & \rho \left(n, \frac{n}{3} \right) &= \frac{n(n-3)(n-6)(3n+29)}{648}, & \rho \left(n, \frac{n}{2} \right) &= \frac{n(n-2)(n-4)(n-6)(37n^2+294n-2320)}{46080}. \end{aligned} \quad (3)$$

n	$d(n)$	$d^*(n)$	n	$d(n)$	$d^*(n)$
7	180	30	19	1 801 329 010	94 806 790
8	3 044	385	20	3 600 529 450	180 028 084
9	26 060	2 900	21	6 925 187 830	329 770 930
10	152 900	15 308	22	12 869 925 310	584 999 362
11	696 905	63 355	23	23 190 544 696	1 008 284 552
12	2 641 925	220 242	24	40 637 416 600	1 693 230 295
13	8 691 683	668 591	25	69 427 501 000	2 777 100 040
14	25 537 655	1 824 311	26	115 901 728 800	4 457 765 752
15	68 396 900	4 559 818	27	189 426 912 675	7 015 811 753
16	169 537 940	10 596 558	28	303 616 322 295	10 843 450 498
17	393 481 660	23 145 980	29	477 960 911 025	16 481 410 725
18	862 928 092	47 941 370	30	739 984 318 125	24 666 159 267

TABLE 1.1. Number $d^*(n)$ of O -topologically non-equivalent functions from the class $C_{1,n-6}(M_3)$

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