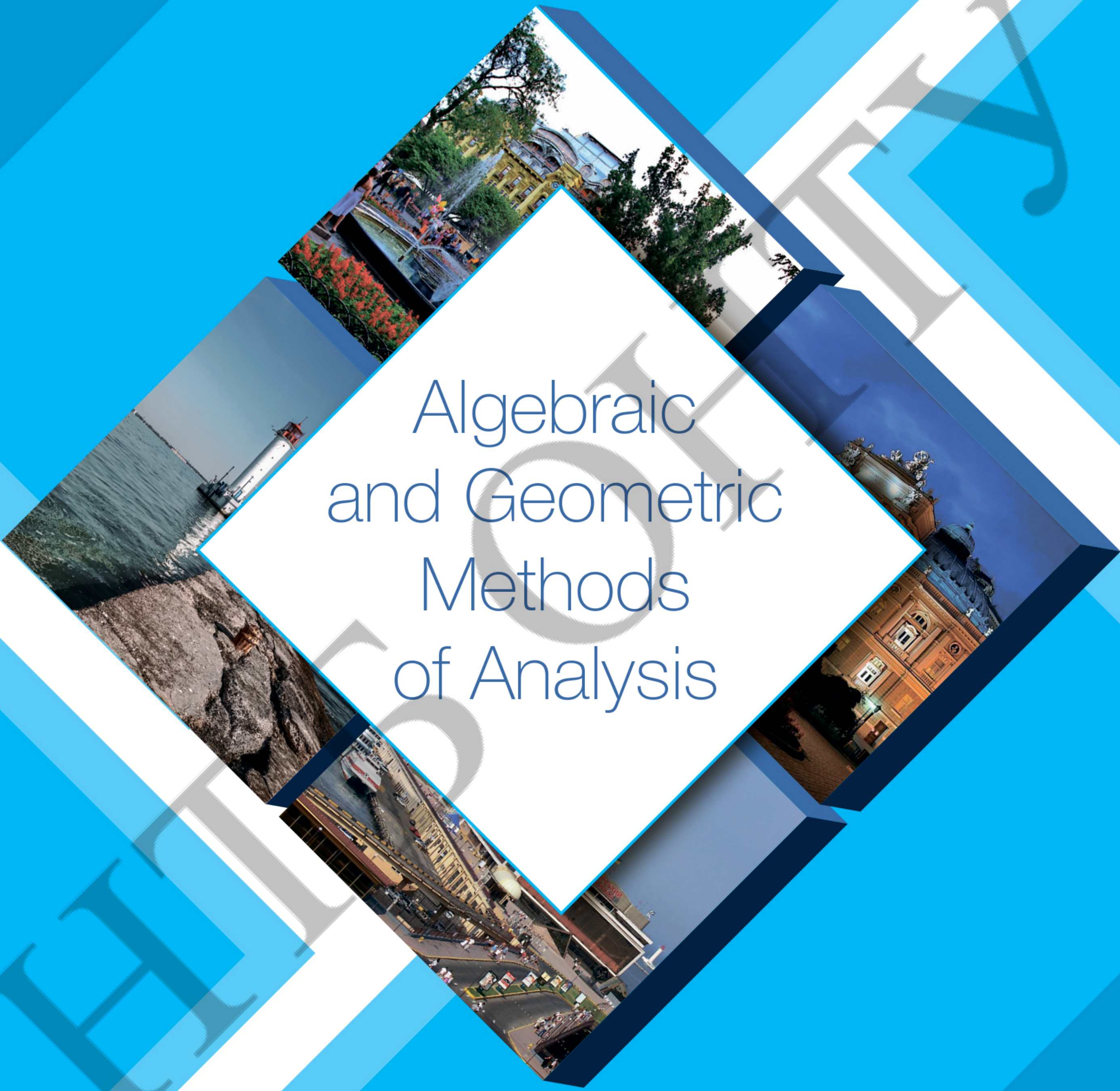
International
Scientific Conference

Algebraic and Geometric Methods of Analysis



27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Group classification of Kolmogorov backward equations with power diffusivity

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We carry out the complete group classification of the class $\mathcal{F}$ of Kolmogorov backward equations with power diffusivity

$$\mathcal{F}_{\alpha\beta}\colon\ \ u_t + xu_y = |x-\alpha|^\beta u_{xx},$$

where α and β are arbitrary real parameters, by solving the same problem for the class $\mathcal{F}'$ of the equations of the form

$$\mathcal{F}'_\beta\colon\quad u_t + xu_y = |x|^\beta u_{xx},$$

with β remains to be the only arbitrary element in the class $\mathcal{F}'$. Using the modified version of the direct method, we compute the equivalence groupoids $\mathcal{G}^\sim_{\mathcal{F}}$ and $\mathcal{G}^\sim_{\mathcal{F}'}$ of the classes $\mathcal{F}$ and $\mathcal{F}'$, respectively, and consequently show that the class $\mathcal{F}'$ is semi-normalized in the usual sense. The modification of the direct method is based on embedding both the classes $\mathcal{F}$ and $\mathcal{F}'$ into the class $\bar{\mathcal{F}}$ of ultraparabolic (1+2)-dimensional Fokker–Planck equations of the form

$$u_t + B(t,x,y)u_y = A^2(t,x,y)u_{xx} + A^1(t,x,y)u_x + A^0(t,x,y)u + C(t,x,y),$$

where the tuple $\bar\theta := (B, A^2, A^1, A^0, C)$ of arbitrary elements of the class $\bar{\mathcal{F}}$ runs through the solution set of the system of the inequalities $A^2 \neq 0$ and $B_x \neq 0$ with no restrictions on A^0, A^1 and C. The equivalence groupoid of of the class $\bar{\mathcal{F}}$ was described in [1, 2] via presenting the equivalence group of this class and stating that it is normalized, see [3] for required notions, results and further references. We use the known determining equations for admissible transformations within the superclass $\bar{\mathcal{F}}$ as the known principal constraints for admissible transformations within the classes $\mathcal{F}$ and $\mathcal{F}'$. After explicitly constructing the groupoids $\mathcal{G}^\sim_{\mathcal{F}}$ and $\mathcal{G}^\sim_{\mathcal{F}'}$, it is easy to show that the group classification of the class $\mathcal{F}$ reduces to that of the class $\mathcal{F}'$.

The class $\mathcal{F}'$ admits a distinguished discrete equivalence transformation

$$\mathscr{J}\colon\quad \tilde t = y\operatorname{sgn}x,\quad \tilde x = \frac{1}{x},\quad \tilde y = t\operatorname{sgn}x,\quad \tilde u = \frac{u}{x},\quad \tilde\beta = 5-\beta,$$

which turns out to be the only point equivalence transformation essential for carrying out the group classification of this class modulo the $\mathcal{G}^\sim_{\mathcal{F}'}$-equivalence.

The following chain of assertions provides the complete solutions to the group classification problems for the classes $\mathcal{F}$ and $\mathcal{F}'$.

Theorem 1. (i) *The point transformations $\mathscr{S}(c_1)$: $(\tilde t, \tilde x, \tilde y, \tilde u, \tilde\beta, \tilde\alpha) = (t, x+c_1, y+c_1t, u, \beta, \alpha+c_1)$ where c_1 is arbitrary constant, constitute a one-parameter group of equivalence transformations of the class $\mathcal{F}$.*

(ii) *The wide family of admissible transformations* $\mathcal{S}_{\alpha\beta} := \big((\alpha,\beta),\pi_*\mathscr{S}(-\alpha),(0,\beta)\big)$ *of the class* $\mathcal{F}$ *from the action groupoid of its equivalence group maps this class onto the class* $\mathcal{F}'$ *interpreted as a subclass of* $\mathcal{F}$.

(iii) *The point transformation* $\mathscr{J}$ *is a (discrete) equivalence transformation of the class* $\mathcal{F}'$.

(iv) *The class* $\mathcal{F}'$ *is semi-normalized with respect to the discrete equivalence subgroup generated by* $\mathscr{J}$. *In other words, the equivalence groupoid* $\mathcal{G}^{\sim}_{\mathcal{F}'}$ *of* $\mathcal{F}'$ *is the Frobenius product of the action groupoid of this subgroup and the fundamental equivalence groupoid* $\mathcal{G}^{\rm f}_{\mathcal{F}'}$ *of* $\mathcal{F}'$.

Corollary 2. (i) *Different equations* $\mathcal{F}'_\beta$ *and* $\mathcal{F}'_{\tilde\beta}$ *are similar with respect to point transformations if and only if* $\beta+\tilde\beta=5$.

(ii) *Equations* $\mathcal{F}_{\alpha\beta}$ *and* $\mathcal{F}_{\tilde\alpha\tilde\beta}$ *are similar with respect to point transformations if and only if either* $\tilde\beta=\beta$ *or* $\beta+\tilde\beta=5$.

(iii) *The equivalence groupoid* $\mathcal{G}^{\sim}_{\mathcal{F}}$ *of* $\mathcal{F}$ *is generated by admissible transformations* $\mathcal{S}_{\alpha\beta}$ *and elements of* $\mathcal{G}^{\sim}_{\mathcal{F}'}$. *More specifically, for each admissible transformation* $\big((\alpha,\beta),\Phi,(\tilde\alpha,\tilde\beta)\big)$ *of* $\mathcal{F}$, *we have* $\Phi=\pi_*\mathscr{S}(\tilde\alpha)\circ\breve\Phi\circ\pi_*\mathscr{S}(-\alpha)$ *for some point transformation* $\breve\Phi$ *with* $(\beta,\breve\Phi,\tilde\beta)\in\mathcal{G}^{\sim}_{\mathcal{F}'}$.

Theorem 3. *The kernel Lie invariance algebra* $\mathfrak{g}^{\cap}_{\mathcal{F}'}$ *of the equations from the class* $\mathcal{F}'$ *is*

$$\mathfrak{g}^{\cap}_{\mathcal{F}'}=\langle\mathcal{P}^t,\mathcal{P}^y,\mathcal{I},(tx-y)\partial_u,x\partial_u,\partial_u\rangle,\quad \textit{where}\quad \mathcal{P}^t:=\partial_t,\quad \mathcal{P}^y:=\partial_y,\quad \mathcal{I}:=u\partial_u.$$

Any equation $\mathcal{F}'_\beta$ *from* $\mathcal{F}'$ *is invariant with respect to the algebra*

$$\mathfrak{g}^{\rm gen}_\beta=\langle\mathcal{P}^t,\mathcal{P}^y,\mathcal{I},\mathcal{D}^\beta,\mathcal{Z}(f^\beta)\rangle\quad \textit{with}\quad \mathcal{D}^\beta:=(2-\beta)t\partial_t+x\partial_x+(3-\beta)y\partial_y,\ \ \mathcal{Z}(f^\beta):=f^\beta\partial_u,$$

where the parameter function $f^\beta=f^\beta(t,x,y)$ *runs through the solution set of this equation, and* $\beta\in(-\infty,5/2]$ *modulo the* $G^{\sim}_{\mathcal{F}'}$*-equivalence. the maximal Lie invariance algebra* $\mathfrak{g}_\beta$ *of the equation* $\mathcal{F}_\beta$ *coincides with* $\mathfrak{g}^{\rm gen}_\beta$ *if and only if* $\beta\in\mathbb{R}\setminus\{0,2,3,5\}$. *A complete list of* $G^{\sim}_{\mathcal{F}'}$*-inequivalent essential Lie symmetry extensions in the class* $\mathcal{F}'$ *is exhausted by the following cases:*

$$\beta=2\colon\quad \mathfrak{g}_2=\mathfrak{g}^{\rm gen}_2\dotplus\langle\mathcal{K}_2\rangle\quad \textit{with}\quad \mathcal{K}_2=2xy\partial_x+y^2\partial_y-xu\partial_u,$$

$$\begin{aligned}\beta=0\colon\quad &\mathfrak{g}_0=\mathfrak{g}^{\rm gen}_0\dotplus\langle\mathcal{K}_0,\mathcal{P}^3,\mathcal{P}^2,\mathcal{P}^1\rangle\quad \textit{with}\\ &\mathcal{K}_0=t^2\partial_t+(tx+3y)\partial_x+3ty\partial_y-(x^2+2t)u\partial_u,\\ &\mathcal{P}^3=3t^2\partial_x+t^3\partial_y+3(y-tx)u\partial_u,\quad \mathcal{P}^2=2t\partial_x+t^2\partial_y-xu\partial_u,\quad \mathcal{P}^1=\partial_x+t\partial_y.\end{aligned}$$

Corollary 4. *The kernel Lie invariance algebra* $\mathfrak{g}^{\cap}_{\mathcal{F}}$ *of the equations from the class* $\mathcal{F}$ *coincides with that for the class* $\mathcal{F}'$, $\mathfrak{g}^{\cap}_{\mathcal{F}}=\mathfrak{g}^{\cap}_{\mathcal{F}'}$ *Any equation* $\mathcal{F}_{\alpha\beta}$ *from* $\mathcal{F}$ *is invariant with respect to the algebra*

$$\mathfrak{g}^{\rm gen}_{\alpha\beta}=\big\langle\mathcal{P}^t,\mathcal{P}^y,\mathcal{I},\mathcal{D}^{\alpha\beta},\mathcal{Z}(f^{\alpha\beta})\big\rangle$$

with $\mathcal{D}^{\alpha\beta}:=(2-\beta)t\partial_t+(x-\alpha)\partial_x+\big((3-\beta)y-\alpha t\big)\partial_y$, $\mathcal{Z}(f^{\alpha\beta}):=f^{\alpha\beta}\partial_u$, *and the parameter function* $f^{\alpha\beta}=f^{\alpha\beta}(t,x,y)$ *running through the solution set of this equation. Modulo the* $\mathcal{G}^{\sim}_{\mathcal{F}'}$*-equivalence, we can assume* $\beta\in(-\infty,5/2]$, *and a complete list of* $\mathcal{G}^{\sim}_{\mathcal{F}'}$*-inequivalent essential Lie symmetry extensions in the class* $\mathcal{F}$ *is exhausted by the counterparts of those in the class* $\mathcal{F}'$, $\mathcal{F}_{00}$ *and* $\mathcal{F}_{02}$. *An analogous list up to the* $G^{\sim}_{\mathcal{F}'}$*-equivalence consists of the equations* $\mathcal{F}_{00}$, $\mathcal{F}_{02}$, $\mathcal{F}_{03}$ *and* $\mathcal{F}_{05}$.

References

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[2] Koval S.D. and Popovych R.O., Extended symmetry analysis of (1+2)-dimensional fine Kolmogorov backward equation, *Stud. Appl. Math.* (2024), https://doi.org/10.1111/sapm.12695, 1–30.

[3] Vaneeva O.O., Bihlo A. and Popovych R.O., Generalization of the algebraic method of group classification with application to nonlinear wave and elliptic equations, *Commun. Nonlinear Sci. Numer. Simul.* **91** (2020), 105419, arXiv:2002.08939.

Integral problem for system of partial differential equations of third order

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Let $H(\mathbb{R}_+ \times \mathbb{R}^n)$ be a class of entire functions on $\mathbb{R}$, K_L is a class of quasipolynomials of the form $\varphi(x) = \sum_{i=1}^{n} Q_r(x)\exp[\alpha_r x]$, where $\alpha_r \in L \subseteq \mathbb{C}$, $\alpha_k \neq \alpha_l$, for $k \neq l$, $Q_r(x)$ are given polynomials.

Each quasipolynomial defines a differential operator $\varphi\left(\frac{\partial}{\partial\lambda}\right)$ of finite order on the class of entire function, in the form $\sum_{r=1}^{m} Q_r\left(\frac{\partial}{\partial\lambda}\right)\exp\left[\alpha_i\frac{\partial}{\partial\lambda}\right]\Big|_{\lambda=0}$.

In the strip $\Omega = \{(t,x)\in\mathbb{R}^{n+1} : t \in \{([T_1,T_2]\cup[T_3,T_4]), x\in\mathbb{R}^n\}$, we consider of the system of equations

$$\frac{\partial^3 U_i}{\partial t^3} + \sum_{j=1}^{n}\left\{a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^2 U_j}{\partial t^2} + b_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial U_j}{\partial t} + c_{ij}\left(\frac{\partial}{\partial x}\right)\right\}U_j(t,x) = 0, \tag{1}$$

$$\int_{T_1}^{T_2} U_{ik}(t,x)dt + \int_{T_3}^{T_4} U_{ik}(t,x)dt = \varphi_{ik}(x), \quad k = 1,2,3, \tag{2}$$

$$\int_{T_1}^{T_2} tU_{ik}(t,x)dt + \int_{T_3}^{T_4} tU_{ik}(t,x)dt = \varphi_{ik}(x). \quad i = 1,...,n, \tag{3}$$

$$\int_{T_1}^{T_2} t^2U_{ik}(t,x)dt + \int_{T_3}^{T_4} t^2U_{ik}(t,x)dt = \varphi_{ik}(x). \tag{4}$$

Where $a_{ij}\left(\frac{\partial}{\partial x}\right), b_{ij}\left(\frac{\partial}{\partial x}\right), c_{ij}\left(\frac{\partial}{\partial x}\right)$, are differential expression with entire symbols $a_{ij}(\lambda)\neq 0, b_{ij}(\lambda)\neq 0, c_{ij}(\lambda)\neq 0$.

Let be $\eta(\lambda) = \int_{T_1}^{T_2} W^{n-1}(t,\lambda)dt + \int_{T_3}^{T_4} W^{n-1}(t,\lambda)dt$ is a certain function $W(t,\lambda)$ is a solution of equation $\left(\frac{d^n}{dt^n} + \sum_{i=1}^{n} a_i(\lambda)\frac{d^{n-i}}{dt^{n-i}}\right)W(t,\lambda) = 0$, satisfies conditions $W^n(t,\lambda)\Big|_{t=0} = 1, \quad W^{n-1}(t,\lambda)\Big|_{t=0} = 0, \quad W(t,\lambda)\Big|_{t=0} = 0.$

Denote be $P = \Big\{\Delta(\lambda) = 0, \lambda\in\mathbb{C}\Big\}$ set zeros of function $\eta(\lambda)$.

Theorem 1. ***Theorem.*** *Let $\varphi_{ik}(x)\in K_L$, $i = 1,...,n$, $j = 1,...,n$ then the class $K_{L\setminus P}$ exist and unique solution of the problem (1)-(4). Solution of the problem (1)-(4) can be represented in the form*

$$U_i(t,x) = \sum_{k=1}^{3}\sum_{p=1}^{n}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)}T_{kjp}(t,\lambda)W(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0},$$

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