

International scientific conference  
**«Algebraic and geometric  
methods of analysis»**

Book of abstracts



May 30 - June 4, 2018,  
Odesa,  
Ukraine

<https://www.imath.kiev.ua/~topology/conf/agma2018>

# The commutator and centralizer of Sylow subgroups of alternating and symmetric groups, its minimal generating set

Ruslan Viacheslavovich Skuratovskii

(Ukraine, Kiev)

*E-mail:* ruslcomp@mail.ru

Given a permutational wreath product sequence of cyclic groups [1, 3] of order 2 we research a commutator width of such groups and some properties of its commutator subgroup. Commutator width of Sylow 2-subgroups of alternating group  $A_{2^k}$ , permutation group  $S_{2^k}$  and  $C_p \wr B$  were founded. The result of research was extended on subgroups  $(Syl_2 A_{2^k})'$ ,  $p > 2$ . The paper presents a construction of commutator subgroup of Sylow 2-subgroups of symmetric and alternating groups. Also minimal generic sets of Sylow 2-subgroups of  $A_{2^k}$  were founded. Elements presentation of  $(Syl_2 A_{2^k})'$ ,  $(Syl_2 S_{2^k})'$  was investigated. We prove that the commutator width [2] of an arbitrary element of a discrete wreath product of cyclic groups  $C_{p_i}$ ,  $p_i \in \mathbb{N}$  is 1.

**Lemma 1.** *For any group  $B$  and integer  $p \geq 2$ ,  $p \in \mathbb{N}$  if  $w \in (B \wr C_p)'$  then  $w$  can be represented as the following wreath recursion*

$$w = (r_1, r_2, \dots, r_{p-1}, r_1^{-1} r_2^{-1} \dots r_{p-1}^{-1} \prod_{j=1}^k [f_j, g_j]),$$

where  $r_1, \dots, r_{p-1}, f_j, g_j \in B$ , and  $k \leq cw(B)$ .

**Lemma 2.** *An element  $(g_1, g_2)\sigma^i \in G'_k$  iff  $g_1, g_2 \in G_{k-1}$  and  $g_1 g_2 \in B'_{k-1}$ .*

**Lemma 3.** *For any group  $B$  and integer  $p \geq 2$  inequality*

$$cw(B \wr C_p) \leq \max(1, cw(B))$$

holds.

**Corollary 4.** *If  $W = C_{p_k} \wr \dots \wr C_{p_1}$  then for  $k \geq 2$   $cw(W) = 1$ .*

**Corollary 5.** *Commutator width  $cw(Syl_p(S_{p^k})) = 1$  for prime  $p$  and  $k > 1$  and commutator width  $cw(Syl_p(A_{p^k})) = 1$  for prime  $p > 2$  and  $k > 1$ .*

**Theorem 6.** *Elements of  $Syl_2 S'_{2^k}$  have the following form  $Syl_2 S'_{2^k} = \{[f, l] \mid f \in B_k, l \in G_k\} = \{[l, f] \mid f \in B_k, l \in G_k\}$ .*

**Theorem 7.** *Commutator width of the group  $Syl_2 A_{2^k}$  equal to 1 for  $k \geq 2$ .*

**Proposition 8.** *The subgroup  $(syl_2 A_{2^k})'$  has a minimal generating set of  $2k - 3$  generators.*

## REFERENCES

- [1] R. Skuratovskii, Generators and relations for Sylows  $p$ -subgroup of group  $S_n$ . (in ukrainian) Naukovi Visti KPI. 2013, N. 4 pp. 94 -105.
- [2] Alexey Muranov, Finitely generated infinite simple groups of infinite commutator width. arXiv:math/0608688v4 [math.GR] 12 Sep 2009.
- [3] R. V. Skuratovskii, Structure and minimal generating sets of Sylow 2-subgroups of alternating groups. Source: <https://arxiv.org/abs/1702.05784v2>

<b>V. Lomadze</b> <i>A purely algebraic construction of Schwartz distributions</i>	<b>34</b>
<b>S. Maksymenko</b> <i>Deformation of functions on orientable surfaces by symplectic diffeomorphisms</i>	<b>36</b>
<b>Martseniuk V., Bondarenko A., Gefter S.</b> <i>Application the p-adic topology on <math>Z</math> for study determinants of infinite order with integer coefficients</i>	<b>37</b>
<b>K. Matsumoto</b> <i>Warped product semi-slant submanifolds in locally conformal Kaehler manifolds, II</i>	<b>38</b>
<b>Mormul P.</b> <i>What points in Goursat Monster Tower are strongly nilpotent</i>	<b>39</b>
<b>Mukhamadiev F. G.</b> <i>The local density and the local weak density of <math>N_7^\varphi</math>-kernel of a topological spaces</i>	<b>40</b>
<b>Tetiana V. Obikhod</b> <i>Topological invariants and microscopic quantum description</i>	<b>41</b>
<b>A. Ozkan</b> <i>On the misconceptions of 10th grade students about analytical geometry</i>	<b>42</b>
<b>Parasyuk I. O.</b> <i>Hyperbolic quasiperiodic motion of charged particle on 2-sphere</i>	<b>43</b>
<b>E. Petrov</b> <i>Representing trees of finite ultrametric spaces and weak similarities</i>	<b>45</b>
<b>Prishlyak A., Hladysh B.</b> <i>Functions with three critical points on closed non-oriented 3-manifolds</i>	<b>47</b>
<b>V.M. Prokip</b> <i>A note on similarity of matrices</i>	<b>49</b>
<b>Romakina L.N.</b> <i>Inversion with respect to an elliptic cycle of a hyperbolic plane of positive curvature</i>	<b>51</b>
<b>Romaniv A. M.</b> <i>On the relationship between the Smith normal forms of matrices and of their least common multiple</i>	<b>53</b>
<b>Ryazanova O.</b> <i>Estimates for the surface with given average curvature</i>	<b>54</b>
<b>Samokhvalov S. E.</b> <i>Kinematic renormalization of energy in the gravity</i>	<b>55</b>
<b>Savchenko A.</b> <i>Fuzzy metrization of the spaces of idempotent measures</i>	<b>56</b>
<b>Sazonova O.</b> <i>General form of the Maxwellian distribution with arbitrary density</i>	<b>57</b>
<b>Skuratovskii R.</b> <i>The commutator and centralizer of Sylow subgroups of alternating and symmetric groups, its minimal generating set</i>	<b>58</b>
<b>Sokhatsky F., Krainichuk H.</b> <i>Semi-lattice of varieties of quasigroups with linearity</i>	<b>59</b>
<b>Salimov R., Stefanchuk M.</b> <i>Asymptotic behavior of solutions to a nonlinear Beltrami equation</i>	<b>61</b>
<b>Mokritskaya T. P., Tushev A. V.</b> <i>On fractal characteristics of the micro-aggregate structure of loess soils</i>	<b>62</b>
<b>Vasilchenko A. N.</b> <i>Spaces of primitive elements in dual modules over Steenrod algebra</i>	<b>64</b>