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Scientific Conference

Algebraic and Geometric Methods of Analysis

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Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

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On the squares of diffeomorphisms of surfaces

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Let M be a surface and $\mathcal{D}(M)$ be the group of C^{∞}-diffeomorphisms of M. There is a natural right action of the group $\mathcal{D}(M)$ on the space of smooth functions $C^{\infty}(M,\mathbb{R})$ defined by the following rule: $(h,f) \longmapsto f \circ h$, where $h \in \mathcal{D}(M)$, $f \in C^{\infty}(M,\mathbb{R})$.

Thus, the *stabilizer* of f with respect to the action

$$\mathcal{S}(f) = \{h \in \mathcal{D}(M) \mid f \circ\ h = f\}$$

consists of f-preserving diffeomorphisms of M.

Endow $\mathcal{D}(M)$ with Whitney C^{∞}-topology and its subspaces $\mathcal{S}(f)$ with induced one. Denote by $\mathcal{S}_{\mathrm{id}}(f)$ the identity path component of $\mathcal{S}(f)$.

Definition 1. Denote by $\mathcal{F}(M)$ the space of smooth functions $f \in C^{\infty}(M,\mathbb{R})$ satisfying the following conditions:

(1) The function f takes constant value at each connected component of ∂M and has no critical points in ∂M.
(2) For every critical point z of f there is a local presentation $f_z\colon \mathbb{R}^2 \to \mathbb{R}$ of f near z such that f_z is a homogeneous polynomial $\mathbb{R}^2 \to \mathbb{R}$ without multiple factors.

Definition 2. A smooth vector field F will be called Hamiltonian-like for $f \in \mathcal{F}(M)$ if the following conditions hold:

(1) $F(x) = 0$ if and only if x is a critical point of f,
(2) f takes constant values on orbits of F,
(3) Let z be a critical point of f. Then there exists a local representation of f at z as a homogeneous polynomial $g\colon (\mathbb{R}^2, 0) \to (\mathbb{R}, 0)$ without multiple factors such that in the same coordinates (x, y) near the origin 0 in $\mathbb{R}^2$ we have $F = -g'_y \frac{\partial}{\partial x} + g'_x \frac{\partial}{\partial y}$.

The smooth flow $\mathbf{F}\colon M \times \mathbb{R} \to M$ generated by a Hamiltonian-like vector field for f will be called Hamiltonian-like flow for f.

Denote by $\Delta^{-}(f)$ the set of diffeomorphisms from $\mathcal{S}(f)$ leaving invariant each regular connected component of each level-set of f and reverses its orientation.

Theorem 3. *Let D^2 be a 2-disk, $f \in \mathcal{F}(M)$. Suppose there exists $h \in \Delta^{-}(f)$, i.e. $\Delta^{-}(f) \neq \varnothing$. Then there exists another $g \in \Delta^{-}(f)$ such that $g = h$ in a neighborhood of ∂D and $g^2 \in \mathcal{S}_{\mathrm{id}}(f)$.*

Theorem 4. *Let M be an orientable connected compact surface and $f \in \mathcal{F}(M)$. If $\Delta^{-}(f) \neq \varnothing$, then there exists another $g \in \Delta^{-}(f)$ such that $g^2 \in \mathcal{S}_{\mathrm{id}}(f)$.*