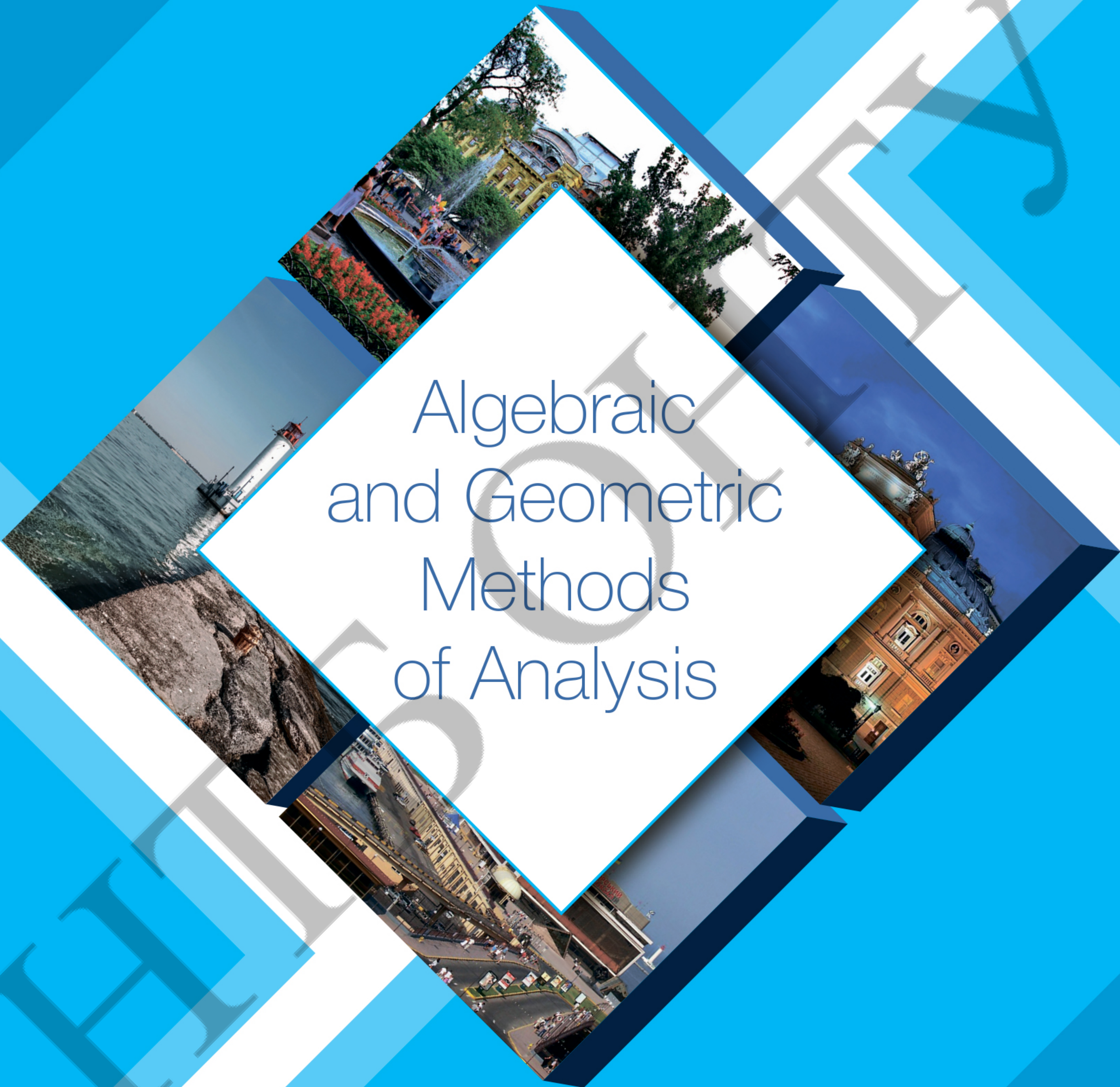
International Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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On singularities of mappings with a Lebesgue integrable majorant

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The following definitions are from [1]. A path γ in $\mathbb{R}^n$ is a continuous mapping $\gamma:\Delta\to\mathbb{R}^n$ where Δ is an interval in $\mathbb{R}$. Its locus $\gamma(\Delta)$ is denoted by $|\gamma|$. Given a family Γ of paths γ in $\mathbb{R}^n$, a Borel function $\rho:\mathbb{R}^n\to[0,\infty]$ is called *admissible* for Γ, abbr. $\rho\in\operatorname{adm}\Gamma$, if

$$\int\limits_{\gamma}\rho(x)|dx|\geqslant 1$$

for each (locally rectifiable) $\gamma\in\Gamma$. Given $p\geqslant 1$, the *p-modulus* of Γ is defined by the relation

$$M_p(\Gamma):=\inf_{\rho\in\operatorname{adm}\Gamma}\int\limits_{\mathbb{R}^n}\rho^p(x)dm(x) \qquad (1)$$

interpreted as $+\infty$ if $\operatorname{adm}\Gamma=\varnothing$.

Given sets E and F and a given domain D in $\overline{\mathbb{R}^n}=\mathbb{R}^n\cup\{\infty\}$, we denote by $\Gamma(E,F,D)$ the family of all paths $\gamma:[0,1]\to\overline{\mathbb{R}^n}$ joining E and F in D, that is, $\gamma(0)\in E$, $\gamma(1)\in F$ and $\gamma(t)\in D$ for all $t\in(0,1)$. Everywhere below, unless otherwise stated, the boundary and the closure of a set are understood in the sense of the extended Euclidean space $\overline{\mathbb{R}^n}$. Let $x_0\in\overline{D}$, $x_0\neq\infty$,

$$S(x_0,r)=\{x\in\mathbb{R}^n:|x-x_0|=r\}\,,S_i=S(x_0,r_i)\,,\quad i=1,2\,,$$

$$A=A(x_0,r_1,r_2)=\{x\in\mathbb{R}^n:r_1<|x-x_0|<r_2\}\,.$$

Let $f:D\to\mathbb{R}^n$, $n\geqslant 2$, and let $Q:\mathbb{R}^n\to[0,\infty]$ be a Lebesgue measurable function such that $Q(y)\equiv 0$ for $y\in\mathbb{R}^n\setminus f(D)$. Let $A=A(y_0,r_1,r_2)$ and $\Gamma_f(y_0,r_1,r_2)$ denotes the family of all paths $\gamma:[a,b]\to D$ such that $f(\gamma)\in\Gamma(S(y_0,r_1),S(y_0,r_2),A(y_0,r_1,r_2))$, i.e., $f(\gamma(a))\in S(y_0,r_1)$, $f(\gamma(b))\in S(y_0,r_2)$, and $f(\gamma(t))\in A(y_0,r_1,r_2)$ for any $a<t<b$. We say that f *satisfies the inverse Poletsky inequality at $y_0\in f(D)$ with respect to p-modulus,* if the relation

$$M_p(\Gamma_f(y_0,r_1,r_2))\leqslant\int\limits_{A}Q(y)\cdot\eta^p(|y-y_0|)\,dm(y) \qquad (2)$$

holds for any $0<r_1<r_2<r_0:=\sup\limits_{y\in f(D)}|y-y_0|$ and any Lebesgue measurable function $\eta:(r_1,r_2)\to[0,\infty]$ such that

$$\int\limits_{r_1}^{r_2}\eta(r)\,dr\geqslant 1\,. \qquad (3)$$

Note that estimates of the type (2) are well known and hold at least for $p=n$ in many classes of mappings (see, e.g., [2, Theorem 3.2], [3, Theorem 6.7.II] and [4, Theorem 8.5]). For $p\neq n$, similar estimates may be found, e.g., in [5] and [6].

A mapping $f : D \to \mathbb{R}^n$ is called *discrete* if the image $\{f^{-1}(y)\}$ of any point $y \in \mathbb{R}^n$ consists of isolated points, and *open* if the image of any open set $U \subset D$ is an open set in $\mathbb{R}^n$.

Later, in the extended space $\overline{\mathbb{R}^n} = \mathbb{R}^n \cup \{\infty\}$ we use the *spherical (chordal) metric* $h(x, y) = |\pi(x) - \pi(y)|$, where π is a stereographic projection of $\overline{\mathbb{R}^n}$ onto the sphere $S^n(\frac{1}{2}e_{n+1}, \frac{1}{2})$ in $\mathbb{R}^{n+1}$, namely,

$$h(x, \infty) = \frac{1}{\sqrt{1 + |x|^2}},$$

$$h(x, y) = \frac{|x - y|}{\sqrt{1 + |x|^2}\sqrt{1 + |y|^2}}, \quad x \neq \infty \neq y \tag{4}$$

(see, e.g., [1, Definition 12.1]). The following statement is true.

Theorem 1. *Let $n \geqslant 2$, $p \geqslant n$, let D be a domain in $\mathbb{R}^n$, $x_0 \in D$, and let $f : D \setminus \{x_0\} \to \mathbb{R}^n$ be an open discrete mapping that satisfies the conditions (2)-(3) at any point $y_0 \in \overline{D'} \setminus \{\infty\}$, where $D' := f(D \setminus \{x_0\})$.*

If $Q \in L^1(D')$, then f has a continuous extension $\overline{f} : D \to \overline{\mathbb{R}^n}$, the continuity of which should be understood in the sense of the chordal metric h in (4). The extended mapping $\overline{f}$ is open and discrete in D. Moreover, if $p = n$ and $\overline{f}(x_0) \neq \infty$, then there is a neighborhood $U \subset D$ of the point x_0 depending only on x_0, and $C = C(n, D, x_0) > 0$ such that

$$|\overline{f}(x) - \overline{f}(x_0)| \leqslant \frac{C_n \cdot (\|Q\|_1)^{1/n}}{\log^{1/n}\left(1 + \frac{\delta}{2|x - x_0|}\right)} \tag{5}$$

for any $x, y \in U$, where $\|Q\|_1$ is the norm of the function Q in $L^1(D')$.

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