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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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Finitely bi-Lipschitz homeomorphisms between Finsler manifolds

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In this talk we investigate the boundary behavior of finitely bi-Lipschitz homeomorphisms between Finsler manifolds. Our study involves the module technique and classes of mappings whose moduli of the curve/surface families are integrally controlled from above and below. The Lusin (N)-property with respect to the k -dimensional Hausdorff measure for the finitely bi-Lipschitz mappings is also established. The talk is based on a joint work with A. Golberg; see [1].

Let \mathbb{M} be an n -dimensional differentiable manifold, $n \geq 2$. By the differentiability we mean C^∞ -differentiability. For a point $x \in \mathbb{M}$, $T_x\mathbb{M}$ denotes the tangent space at x , and $T\mathbb{M} := \cup_{x \in \mathbb{M}} T_x\mathbb{M}$ is the tangent bundle. The *Finsler manifold* is a differentiable manifold \mathbb{M} equipped the Finsler metric $\Phi(x, \xi) : T\mathbb{M} \rightarrow \mathbb{R}^+$ satisfying the conditions:

- (i) *regularity*: $\Phi \in C^\infty$ on $T\mathbb{M}_0 := T\mathbb{M} \setminus \{0\}$;
- (ii) *positive homogeneity*: Φ is positive homogeneous that is $\Phi(x, a\xi) = a\Phi(x, \xi)$ for all positive $a \in \mathbb{R}$ and $\Phi(x, \xi) > 0$ for $\xi \neq 0$;
- (iii) *the Legendre condition or strong convexity condition*: $g_{ij}(x, \xi) = \frac{1}{2} \frac{\partial^2 \Phi^2(x, \xi)}{\partial \xi^i \partial \xi^j}$ is positive definite whenever $\xi \neq 0$.

Following [3], an element of *volume* on the Finsler manifold is defined by $d\sigma_\Phi(x) := \frac{|B^n|}{|B_x^n|} dx^1 \dots dx^n$, where $|B^n|$ denotes the Euclidean volume of the unit n -ball whereas $|B_x^n|$ is the Euclidean volume of the set $B_x^n = \left\{ (\xi^1, \dots, \xi^n) \in \mathbb{R}^n : \Phi \left(x, \sum_{i=1}^n (\xi^i, e_i(x)) \right) < 1 \right\}$ with an arbitrary basis $\{e_i(x)\}_{i=1}^n$ in \mathbb{R}^n depending on x .

Suppose that D and D' are two domains on \mathbb{M} and \mathbb{M}' , respectively, $f : D \rightarrow D'$ is a continuous mapping. Let $L(x, f) = \limsup_{y \rightarrow x} \frac{d_{\Phi'}(f(x), f(y))}{d_\Phi(x, y)}$, $x \in D$ and $l(x, f) = \liminf_{y \rightarrow x} \frac{d_{\Phi'}(f(x), f(y))}{d_\Phi(x, y)}$.

Following [2], we say that $f : D \rightarrow D'$ is *finitely Lipschitz* if $L(x, f) < \infty$ for all $x \in D$ and *finitely bi-Lipschitz* if

$$0 < l(x, f) \leq L(x, f) < \infty$$

for all $x \in D$.

A Borel function $\rho : \mathbb{M} \rightarrow [0, \infty]$ is called *admissible* for the family Γ of k -dimensional surfaces S in \mathbb{M} , $k = 1, \dots, n-1$, (abbr. $\rho \in \text{adm } \Gamma$), if

$$\int_S \rho^k dA_\Phi \geq 1, \quad \forall S \in \Gamma. \quad (1)$$

Following [2], the function $\rho : \mathbb{M} \rightarrow [0, \infty]$ measurable with respect to the measure of a volume σ_Φ is called *extensively admissible* for a family Γ of k -dimensional surfaces S in \mathbb{M} (abbr. $\rho \in \text{ext adm } \Gamma$), if the admissibility condition (1) holds for almost all (a.a.) $S \in \Gamma$.

The *conformal module* or *module* (called also the conformal modulus) of a family Γ of k -dimensional surfaces in D is defined by

$$M(\Gamma) := \inf_{\rho \in \text{adm } \Gamma} \int_D \rho^n(x) d\sigma_\Phi(x),$$

where D is a domain in \mathbb{M} .

Let $Q : \mathbb{M} \rightarrow (0, \infty)$ be a measurable function. A homeomorphism $f : D \rightarrow D'$ is called *lower Q -homeomorphism* at a point $x_0 \in \overline{D}$, if there exists $\delta_0 \in (0, d(x_0))$, $d(x_0) := \sup_{x \in D} d_{\Phi}(x, x_0)$, such that for any $\varepsilon_0 < \delta_0$ and any geodesic rings $A_{\varepsilon} = A(x_0, \varepsilon, \varepsilon_0) = \{x \in \mathbb{M} : \varepsilon < d_{\Phi}(x, x_0) < \varepsilon_0\}$, $\varepsilon \in (0, \varepsilon_0)$, the inequality

$$M(f(\Sigma_{\varepsilon})) \geq \inf_{\rho \in \text{ext adm } \Sigma_{\varepsilon}} \int_{D \cap A_{\varepsilon}} \frac{\rho^n(x)}{Q(x)} d\sigma_{\Phi}(x) \quad (2)$$

holds. Here Σ_{ε} stands for the family of all intersections of the geodesic spheres $S(x_0, r) = \{x \in \mathbb{M} : d_{\Phi}(x, x_0) = r\}$, $r \in (\varepsilon, \varepsilon_0)$, with the domain D . We say that the homeomorphism $f : D \rightarrow D'$ is a *lower Q -homeomorphism* in D , if f is lower Q -homeomorphism at every point $x_0 \in \overline{D}$.

For sets A, B and C , we denote by $\Delta(A, B; C)$ the set of all curves $\gamma : [a, b] \rightarrow \mathbb{M}$, which join A and B in C , i.e. $\gamma(a) \in A$, $\gamma(b) \in B$ and $\gamma(t) \in C$ for all $t \in (a, b)$.

Let $Q : \mathbb{M} \rightarrow (0, \infty)$ be a measurable function. We say that a homeomorphism $f : D \rightarrow D'$ is *ring Q -homeomorphism at a point $x_0 \in \overline{D}$* , if

$$M(\Delta(f(K), f(K_0); D')) \leq \int_{D \cap A_{\varepsilon}} Q(x) \cdot \eta^n(d_{\Phi}(x, x_0)) d\sigma_{\Phi}(x) \quad (3)$$

holds for any geodesic ring $A_{\varepsilon} = A(x_0, \varepsilon, \varepsilon_0)$, $0 < \varepsilon < \varepsilon_0 < \infty$, any two continua (compact connected sets) $K \subset \overline{B(x_0, \varepsilon)} \cap D$ and $K_0 \subset D \setminus B(x_0, \varepsilon_0)$ and each Borel function $\eta : (\varepsilon, \varepsilon_0) \rightarrow [0, \infty]$, such that $\int_{\varepsilon}^{\varepsilon_0} \eta(r) dr = 1$. We say that f is a *ring Q -homeomorphism in D* , if (3) holds for all points $x_0 \in \overline{D}$.

Recall that a metric space \mathbb{M} is called *hyperconvex* if $\bigcap_{\alpha \in \Lambda} \overline{B}(x_{\alpha}, r_{\alpha}) \neq \emptyset$ for any collection of points $\{x_{\alpha}\}_{\alpha \in \Lambda}$ in \mathbb{M} and positive numbers $\{r_{\alpha}\}_{\alpha \in \Lambda}$ such that $d(x_{\alpha}, x_{\beta}) \leq r_{\alpha} + r_{\beta}$ for any α and β in Λ .

The main result of talk is following

Theorem 1. ([1]) *Let D and D' be two domains in Finsler n -dimensional manifolds (\mathbb{M}, Φ) and (\mathbb{M}', Φ') , respectively, $n \geq 2$, and let \mathbb{M}' be a hyperconvex space. If $f : D \rightarrow D'$ is a finitely bi-Lipschitz homeomorphism then f is both lower Q -homeomorphism with $Q = K_I^{\frac{1}{n-1}}(x, f)$ and ring Q_* -homeomorphism with $Q_* = C \cdot K_I(x, f)$, where $K_I(x, f) \in L_{loc}^1$ stands for the inner dilatation of mapping f , and C is a constant arbitrarily close to 1.*

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Зміст

E. Afanas'eva <i>Finitely bi-Lipschitz homeomorphisms between Finsler manifolds</i>	3
Aliyev Yagub <i>About longest and shortest chords passing through a fixed point</i>	5
S. Antonyan <i>Some equivariant properties of Milnor's construction</i>	6
K. Antoshyna, S. Kozerenko <i>Commuting sets for topological set operators</i>	7
B. Apanasov <i>Asymptotic analysis of quasi-regular mappings in space</i>	8
M. J. Atteya <i>Generalized (σ, τ)-derivations on associative rings satisfying certain identities</i>	10
V. Balan <i>The Tucker HO-SVD and the anisotropy of Finslerian geometric models</i>	11
V. Balashchenko, D. Vylegzhanin <i>Invariant structures on homogeneous Φ-spaces and Lie groups</i>	13
T. Banakh <i>Every 2-dimensional Banach space has the Mazur-Ulam property</i>	15
A. Bandura, V. Baksa, O. Skaskiv <i>A connection between L-index of vector-valued entire function and L-index of each its component</i>	16
B. Baratov, Yu. Eshkabilov <i>Separable cubic stochastic operators</i>	18
V. Bilet, O. Dovgoshey <i>Asymptotically equivalent subspaces of metric spaces</i>	20
E. Bonacci <i>Isomorphic issues about the CTCs in Quantum Physics</i>	22
P. Petrenko, A. Andreev <i>Geometrical Langlands Ramifications and Differential Operators Classification by Verma Module Extensions</i>	23
Y. Cherevko, V. Berezovski, J. Mikeš, Y. Fedchenko <i>Conharmonic Transformations of Locally Conformal Kähler Manifolds</i>	24
V. Chernov <i>Applications of Linking to the Study of Causality</i>	26
A. Bakhtin, I. Denega <i>Problem on extremal decomposition of the complex plane</i>	27
A. Dikarev, A. S. Galaev <i>Parallel spinors on Lorentzian Weyl spaces</i>	29
Yu. A. Drozd <i>Matrix problems, triangulated categories and stable homotopy types</i>	30
V. S. Dryuma <i>On the properties smooth manifolds defined by intersections</i>	31
K. Eftekharinasab <i>Some applications of transversality for infinite dimensional manifolds</i>	33
S. Favorov <i>Uniqueness theorems for almost periodic objects</i>	34
V. Fedorchuk, V. Fedorchuk <i>On symmetry reduction and some classes of invariant solutions of the $(1 + 3)$-dimensional homogeneous Monge-Ampère equation</i>	35
B. Feshchenko <i>Deformations of circle-valued Morse functions on 2-torus</i>	37