International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of

Dvytro Grave

(25.08.1863 - 19.12.1939)

Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023

Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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In our considerations we have direct systems of Minkowski balls, Minkowski domains and direct systems of critical lattices with respective maps and homomorphisms. Let $\mathbb{Q}_2$ and $\mathbb{Z}_2$ be respectively the field of 2-adic numbers and its ring of integers. Denote the corresponding direct limits by D_p^{dirlim} and by Λ_p^{dirlim}.

Proposition 6. $D_p^{dirlim} = \varinjlim 2^m D_p \in (\mathbb{Q}_2/\mathbb{Z}_2)D_p = (\bigcup_m \frac{1}{2^m}\mathbb{Z}_2/\mathbb{Z}_2)D_p$.

Proposition 7. $\Lambda_p^{dirlim} = \varinjlim 2^m \Lambda_p \in (\mathbb{Q}_2/\mathbb{Z}_2)\Lambda_p = (\bigcup_m \frac{1}{2^m}\mathbb{Z}_2/\mathbb{Z}_2)\Lambda_p$.

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On KB(Kantorovich-Banach) spaces and KB operators

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Let E be a Banach lattice and X be a Banach space. E is said to be a KB space if a positive increasing sequence in the closed unit ball of E converges. Every KB-space has order continuous norm, but the converse is not true in general. c_0 has order continuous norm, but c_0 is not a KB-space. For $1 \le p < \infty$, L^p-spaces are KB-spaces.

An operator $T : E \to X$ is said to be a KB operator if for every positive increasing sequence (x_n) in the closed unit ball of E , the sequence (Tx_n) converges. An operator $T : X \to X$ is called demicompact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ converges to $x \in X$, there is a convergent subsequence of (x_n). An operator $T : X \to X$ is said to be a demi Dunford-Pettis if, for every sequence (x_n) in X such that (x_n) converges to zero weakly and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$. Every Dunford-Pettis operator is demi Dunford-Pettis operator. An operator $T : E \to E$ is called a demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is norm convergent to $x \in E$, there is a norm convergent subsequence of (x_n). For the identity operator $I : E \to E$, the operator $2I$ is a demi KB-operator. Every KB operator is a demi KB operator.

Definition 1. Let E be a Banach lattice. An operator $T : E \to E$ is said to be an unbounded demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is unbounded norm convergent to $x \in E$, there is an unbounded norm convergent subsequence of (x_n).

Theorem 2. *Let E be a Banach lattice. Every KB operator $T : E \to E$ is unbounded demi KB operator.*

In this study, we characterize the operators on Banach lattices that under which conditions they satisfy unbounded demi KB operators.

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On polynomial and regular maps of spheres

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This talk offers some results on to the intersection of algebraic topology and algebraic geometry.

Let K be a field and $X \subseteq K^m$, $Y \subseteq K^n$ algebraic sets. Recall that a map $f = (f_1, \ldots, f_n) : X \to Y$ is called *polynomial* (resp. *regular*) if there are polynomials $F_i, G_i \in \mathbb{R}[X_1, \ldots, X_m]$ such that $f_i(x) = F_i(x)$ (resp. $f_i(x) = \frac{F_i(x)}{G_i(x)}$, $G_i(X) \neq 0$) with $i = 1, \ldots, n$ for $x \in X$.

Remark 1. If K is a algebraically closed field then the only regular maps of algebraic sets are polynomial maps.

Example 2. (1) Let $K = \mathbb{R}$ or $\mathbb{C}$, the fields of reals or complex numbers. The n-sphere

$$\mathbb{S}^n(K) = \{(x_1, \ldots, x_{n_1}) \in \mathbb{K}^{n+1};\ x_1^2 + \cdots + x_{n+1}^2 = 1\} = V(X_0^2 + \cdots + X_n^2 - 1)$$

is an algebraic set in $\mathbb{K}^{n+1}$. Write $\mathbb{S}^n(\mathbb{R}) = \mathbb{S}^n$ and notice a diffeomorphism $\mathbb{S}^n(\mathbb{C}) \approx T\mathbb{S}^n$, the tangent bundle of $\mathbb{S}^n$. Consequently, a homotopy equivalence $\mathbb{S}^n(\mathbb{C}) \simeq \mathbb{S}^n$.

(2) Let $K = \mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ with the skew $\mathbb{R}$-algebra $\mathbb{H}$ of quaternions. The Grassmannian (of r-planes in K^n), can be identified with $G_{n,r}(K) = \{A \in M_n(K);\ A^2 = A,\ \bar{A} = A^t,\ \mathrm{rk}(A) = r\}$ for the set $M_n(K)$ of all $n \times n$-matrices over K.

But, for any idempotent $n \times n$ matrix over K, its rank coincides with the trace. Therefore, $G_{n,r}(K)$ can be viewed as a real affine variety.