



International
Scientific Conference



Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

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Odesa, Ukraine

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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In our considerations we have direct systems of Minkowski balls, Minkowski domains and direct systems of critical lattices with respective maps and homomorphisms. Let \mathbb{Q}_2 and \mathbb{Z}_2 be respectively the field of 2-adic numbers and its ring of integers. Denote the corresponding direct limits by D_p^{dirlim} and by Λ_p^{dirlim} .

Proposition 6. $D_p^{dirlim} = \varinjlim 2^m D_p \in (\mathbb{Q}_2/\mathbb{Z}_2)D_p = (\bigcup_m \frac{1}{2^m} \mathbb{Z}_2/\mathbb{Z}_2)D_p$.

Proposition 7. $\Lambda_p^{dirlim} = \varinjlim 2^m \Lambda_p \in (\mathbb{Q}_2/\mathbb{Z}_2)\Lambda_p = (\bigcup_m \frac{1}{2^m} \mathbb{Z}_2/\mathbb{Z}_2)\Lambda_p$.

REFERENCES

- [1] H. Minkowski, *Diophantische Approximationen*, Leipzig: Teubner (1907).
- [2] L.J. Mordell, Lattice points in the region $|Ax^4 + By^4| \leq 1$, *J. London Math. Soc.* **16** (1941), 152–156.
- [3] C. Davis, Note on a conjecture by Minkowski, *J. London Math. Soc.*, **23**, 172–175 (1948).
- [4] H. Cohn, Minkowski's conjectures on critical lattices in the metric $\{|\xi|^p + |\eta|^p\}^{1/p}$, *Annals of Math.*, **51**, (2), 734–738 (1950).
- [5] G. Watson, Minkowski's conjecture on the critical lattices of the region $|x|^p + |y|^p \leq 1$, (I), (II), *Jour. London Math. Soc.*, **28**, (3, 4), 305–309, 402–410 (1953).
- [6] J. W. S. Cassels, *An Introduction to the Geometry of Numbers*, Springer, NY (1997).
- [7] L.S. Pontryagin, *Select Works Volume 1*, CRC Press, Boca Raton London NY (2019).
- [8] N. Glazunov, A. Golovanov, A. Malyshev, Proof of Minkowski's hypothesis about the critical determinant of $|x|^p + |y|^p < 1$ domain, *Research in Number Theory* **9**. Notes of scientific seminars of LOMI. **151**(1986), Nauka, Leningrad, 40–53.
- [9] N. Glazunov, On packing of Minkowski balls, *Comptes rendus de l'Académie bulgare Sci.*, Tome 76, No 3 (2023), 335-342.

On KB(Kantorovich-Banach) spaces and KB operators

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Let E be a Banach lattice and X be a Banach space. E is said to be a KB space if a positive increasing sequence in the closed unit ball of E converges. Every KB -space has order continuous norm, but the converse is not true in general. c_0 has order continuous norm, but c_0 is not a KB -space. For $1 \leq p < \infty$, L^p -spaces are KB -spaces.

An operator $T : E \rightarrow X$ is said to be a KB operator if for every positive increasing sequence (x_n) in the closed unit ball of E , the sequence (Tx_n) converges. An operator $T : X \rightarrow X$ is called demicontact if, for every bounded sequence (x_n) in X such that $(x_n - Tx_n)$ converges to $x \in X$, there is a convergent subsequence of (x_n) . An operator $T : X \rightarrow X$ is said to be a demi Dunford-Pettis if, for every sequence (x_n) in X such that (x_n) converges to zero weakly and $\|x_n - Tx_n\| \rightarrow 0$ as $n \rightarrow \infty$, we have $\|x_n\| \rightarrow 0$ as $n \rightarrow \infty$. Every Dunford-Pettis operator is demi Dunford-Pettis operator. An operator $T : E \rightarrow E$ is called a demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is norm convergent to $x \in E$, there is a norm convergent subsequence of (x_n) . For the identity operator $I : E \rightarrow E$, the operator $2I$ is a demi KB -operator. Every KB operator is a demi KB operator.

Definition 1. Let E be a Banach lattice. An operator $T : E \rightarrow E$ is said to be an unbounded demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is unbounded norm convergent to $x \in E$, there is an unbounded norm convergent subsequence of (x_n) .

Theorem 2. Let E be a Banach lattice. Every KB operator $T : E \rightarrow E$ is unbounded demi KB operator.

In this study, we characterize the operators on Banach lattices that under which conditions they satisfy unbounded demi KB operators.

REFERENCES

- [1] C.D. Aliprantis, O. Burkinshaw. *Positive Operators*, Academic Press, London, 1985.
- [2] H. Benkhaled, A. Jeribi, The class of demi KB - operators on Banach lattices, *Turkish J. Math.*, **47**, 387-396, 2023.
- [3] Y.A. Dabborasad, E.Y. Emelyanov, M.A.A. Marabeh, $u\tau$ -convergence in locally solid vector lattices, *Positivity*, **22**, 1065-1080, 2018.
- [4] P. Meyer-Nieberg, *Banach Lattices*, Springer-Verlag, New York, 1991.
- [5] W.V. Petryshyn, Construction of fixed points of demicompact mappings in Hilbert space, *J. Math. Anal. Appl.*, **14**, 276-284, 1966.

On polynomial and regular maps of spheres

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This talk offers some results on to the intersection of algebraic topology and algebraic geometry.

Let K be a field and $X \subseteq K^m$, $Y \subseteq K^n$ algebraic sets. Recall that a map $f = (f_1, \dots, f_n) : X \rightarrow Y$ is called *polynomial* (resp. *regular*) if there are polynomials $F_i, G_i \in \mathbb{R}[X_1, \dots, X_m]$ such that $f_i(x) = F_i(x)$ (resp. $f_i(x) = \frac{F_i(x)}{G_i(x)}$, $G_i(x) \neq 0$) with $i = 1, \dots, n$ for $x \in X$.

Remark 1. If K is a algebraically closed field then the only regular maps of algebraic sets are polynomial maps.

Example 2. (1) Let $K = \mathbb{R}$ or \mathbb{C} , the fields of reals or complex numbers. The n -sphere

$$\mathbb{S}^n(K) = \{(x_1, \dots, x_{n+1}) \in \mathbb{K}^{n+1}; x_1^2 + \dots + x_{n+1}^2 = 1\} = V(X_0^2 + \dots + X_n^2 - 1)$$

is an algebraic set in \mathbb{K}^{n+1} . Write $\mathbb{S}^n(\mathbb{R}) = \mathbb{S}^n$ and notice a diffeomorphism $\mathbb{S}^n(\mathbb{C}) \approx T\mathbb{S}^n$, the tangent bundle of \mathbb{S}^n . Consequently, a homotopy equivalence $\mathbb{S}^n(\mathbb{C}) \simeq \mathbb{S}^n$.

(2) Let $K = \mathbb{R}, \mathbb{C}, \mathbb{H}$ with the skew \mathbb{R} -algebra \mathbb{H} of quaternions. The Grassmannian (of r -planes in K^n), can be identified with $G_{n,r}(K) = \{A \in M_n(K); A^2 = A, \bar{A} = A^t, \text{rk}(A) = r\}$ for the set $M_n(K)$ of all $n \times n$ -matrices over K .

But, for any idempotent $n \times n$ matrix over K , its rank coincides with the trace. Therefore, $G_{n,r}(K)$ can be viewed as a real affine variety.

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