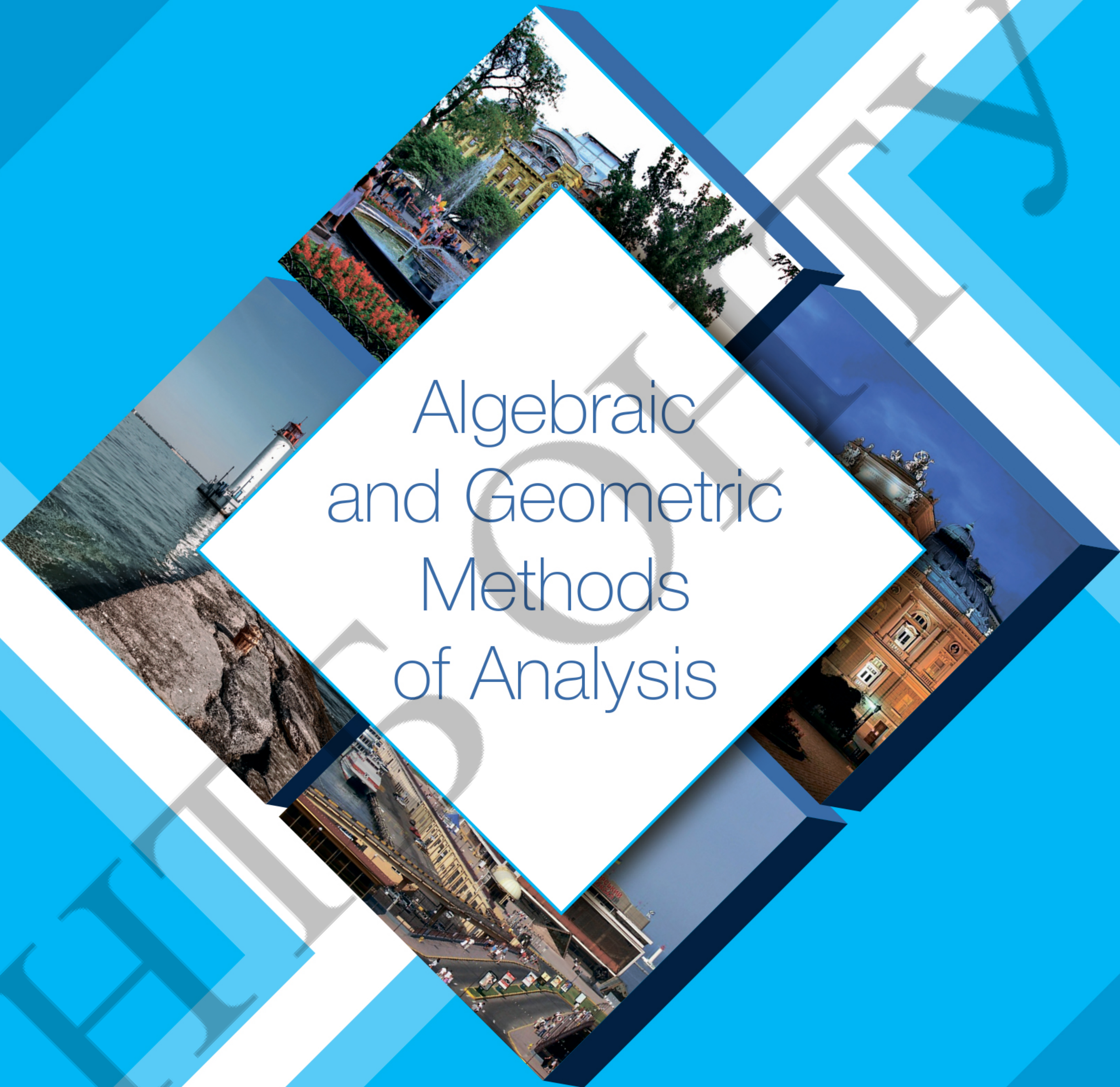
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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generate an associative subalgebra of $\mathbb{O}$. Then, we have a Stiefel map $\pi : V'_{n,r}(\mathbb{O}) \to G_{n,r}(\mathbb{O})$ given by $\pi(A) = A\bar{A}^t$.

Similarly to $V'_{n,r}(\mathbb{O})$, we define $G'_{n,r}(\mathbb{O})$ as follows: $A \in G'_{n,r}(\mathbb{O})$ if all entries of A generate an associative subalgebra of $\mathbb{O}$. It is clear that the Stiefel map $\pi : V'_{n,r}(\mathbb{O}) \to G_{n,r}(\mathbb{O})$ yields a surjective map $\pi : V'_{n,r}(\mathbb{O}) \to G'_{n,r}(\mathbb{O})$. In particular, $G'_{n,r}(\mathbb{O})$ is piecewise-smooth path-connected.

Vector bundles

By analogy with real, complex or quaterionic vector bundles, define an octonionic vector bundle of rank r over some space X as a continuous map

$$f : X \to G'_{-,r}(\mathbb{O}).$$

The following result holds by adapting proof of [1, Theorem 12.1.7] or [2, Theorem 2.5]:

Theorem 2. *Any smooth map $X \to G'_{n,r}(\mathbb{O})$ with $r \leq n$ and X, a compact non-singular algebraic set, is homotopic to an entire rational map from X to $G'_{n,r}(\mathbb{O})$.*

Then, we derive:

Corollary 3. *Any smooth map $X \to G_{n,r}(\mathbb{O})$ with $n = 2, 3$ and $r = 1$ is homotopic to an entire rational map $X \to G_{n,r}(\mathbb{O})$. In particular, any continuous map $\mathbb{S}^n \to \mathbb{S}^8$ is homotopic to an entire rational map $\mathbb{S}^n \to \mathbb{S}^8$.*

References

[1] J. Bochnak, M. Coste, and M.F. Roy, "Real Algebraic Geometry", Ergeb. Math. **36**, Springer-Verlag, Berlin, Heidelberg, New York (1998).

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[3] M. Golasiński and F. Gómez Ruiz, "Grassmann and Stiefel varieties over composition algebras", Springer-Verlag, RSME Springer Series **9**, (2023).

Some series involving central binomial coefficients

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Using Maclaurin expansion $\arcsin = \sum_{n=0}^{\infty} \frac{1}{2^{2n}}\binom{2n}{n}\frac{x^{2n+1}}{2n+1}$ and, for non-zero real variable x, formulas

$$\Re\Big(\arcsin\big(x\sqrt{i}\,\big)\Big) = \arctan\sqrt{\frac{\sqrt{1+x^4}-1}{x^2}},$$

$$\Im\Big(\arcsin\big(x\sqrt{i}\,\big)\Big) = \operatorname{arctanh}\sqrt{\frac{\sqrt{1+x^4}-1}{x^2}},$$

we obtain some series involving central binomial coefficients $\binom{2n}{n}$; see [1] for more details.

Theorem 1. *For $|x| \leq 1$, we have*

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{4^n(2n+1)}\binom{2n}{n} x^{2n+1} = \sqrt{2}\arctan\Big(\frac{\sqrt{\sqrt{1+x^4}-1}}{x}\Big),$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{4^n} \binom{2n}{n} x^{2n} = \frac{\sqrt{2\sqrt{1+x^4}-2}}{\sqrt{1+x^4}\bigl(x^2-1+\sqrt{1+x^4}\bigr)},$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor} n}{4^n} \binom{2n}{n} x^{2n} = \frac{x^2}{\sqrt{2}} \cdot \frac{3x^6-4x^4+5x^2-2+(3x^4-5x^2+2)\sqrt{1+x^4}}{(\sqrt{1+x^4}+x^2-1)^2\sqrt{(1+x^4)^3}\sqrt{\sqrt{1+x^4}-1}}.$$

Example 2. If $x=1$, $x=\sqrt{2}/2$, and $x=1/2$ then from Theorem 1 we have

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{4^n(2n+1)} \binom{2n}{n} = \sqrt{2}\operatorname{arccot}\sqrt{\delta}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{4^n} \binom{2n}{n} = \frac{1}{\sqrt{2\delta}},$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor} n}{4^n} \binom{2n}{n} = -\frac{\sqrt{\delta}}{4};$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{8^n(2n+1)} \binom{2n}{n} = 2\operatorname{arccot}\bigl(\alpha\sqrt{\alpha}\bigr), \qquad \sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{8^n} \binom{2n}{n} = \frac{2}{\sqrt{5\alpha}},$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor} n}{8^n} \binom{2n}{n} = -\frac{\sqrt{5}}{25}\alpha^2\sqrt{\alpha};$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{16^n(2n+1)} \binom{2n}{n} = 2\sqrt{2}\operatorname{arccot}\sqrt{\sqrt{17}+4}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{16^n} \binom{2n}{n} = \frac{2}{\sqrt{17}}\sqrt{\sqrt{17}-1},$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor} n}{16^n} \binom{2n}{n} = -\frac{1}{17\sqrt{17}}\sqrt{17\sqrt{17}+47},$$

where $\alpha=(1+\sqrt{5})/2$ and $\delta=\sqrt{2}+1$ are the golden and silver ratios, respectively.

We will also establish connections with the Fibonacci and Lucas numbers. As usual, the Fibonacci numbers F_n and the Lucas numbers L_n are defined, for $n\in\mathbb{Z}$, through the recurrence $F_n=F_{n-1}+F_{n-2}$, $n\geq 2$, with initial values $F_0=0$, $F_1=1$ and $L_n=L_{n-1}+L_{n-2}$ with $L_0=2$, $L_1=1$. For negative subscripts, we have $F_{-n}=(-1)^{n-1}F_n$ and $L_{-n}=(-1)^n L_n$.

Theorem 3. *For any integer s,*

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{16^n(2n+1)} \binom{2n}{n} F_{2n+s} = -\frac{2\sqrt{10}}{5}\bigl(\alpha^{s-1}\arctan C_1 - \beta^{s-1}\arctan D_1\bigr),$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{16^n(2n+1)} \binom{2n}{n} L_{2n+s} = -2\sqrt{2}\bigl(\alpha^{s-1}\arctan C_1 + \beta^{s-1}\arctan D_1\bigr);$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{16^n} \binom{2n}{n} F_{2n+s} = \frac{8\sqrt{205}}{615}\,(\alpha^s C_2 + \beta^s D_2),$$

$$\sum_{n=0}^{\infty} \frac{(-1)^{\lfloor 3n/2 \rfloor}}{16^n} \binom{2n}{n} L_{2n+s} = \frac{40\sqrt{41}}{615}\,(\alpha^s C_2 - \beta^s D_2);$$

$$\sum_{n=0}^{\infty} (-1)^{\lfloor 3n/2 \rfloor}\frac{n}{16^n} \binom{2n}{n} F_{2n+s} = \frac{\sqrt{205}}{226935}\bigl(\alpha^{s+2}C_3 - \beta^{s+2}D_3\bigr),$$

$$\sum_{n=0}^{\infty} (-1)^{\lfloor 3n/2 \rfloor}\frac{n}{16^n} \binom{2n}{n} L_{2n+s} = \frac{\sqrt{41}}{45387}\bigl(\alpha^{s+2}C_3 + \beta^{s+2}D_3\bigr),$$

where

$$C_1=\beta\sqrt{\frac{\sqrt{78+6\sqrt{5}}-8}{2}},\quad D_1=\alpha\sqrt{\frac{\sqrt{78-6\sqrt{5}}-8}{2}},$$

$$C_2=\frac{\sqrt{\sqrt{78+6\sqrt{5}}-8}\,\sqrt{78-6\sqrt{5}}}{-5+\sqrt{5}+\sqrt{78+6\sqrt{5}}},\quad D_2=\frac{\sqrt{\sqrt{78-6\sqrt{5}}-8}\,\sqrt{78+6\sqrt{5}}}{5+\sqrt{5}-\sqrt{78-6\sqrt{5}}},$$

$$C_3=\frac{\big(148-112\sqrt{5}-\sqrt{78+6\sqrt{5}}\,(25-11\sqrt{5})\big)\sqrt{(78-6\sqrt{5})^3}}{\big(-5+\sqrt{5}+\sqrt{78+6\sqrt{5}}\big)^2\sqrt{\sqrt{78+6\sqrt{5}}-8}},$$

$$D_3=\frac{\big(148+112\sqrt{5}-\sqrt{78-6\sqrt{5}}\,(25+11\sqrt{5})\big)\sqrt{(78+6\sqrt{5})^3}}{\big(5+\sqrt{5}-\sqrt{78+6\sqrt{5}}\big)^2\sqrt{\sqrt{78-6\sqrt{5}}-8}}.$$

Note that since $\binom{2n}{n}=(n+1)C_n$, where C_n are Catalan numbers, our results could be stated equivalently in terms of the Catalan numbers. Similar series were studied recently in [2, 3, 4, 5].

References

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On non-topologizable semigroups

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In this paper we shall follow the semigroup terminology of [1, 2, 3, 4].

Throughout these abstract we always assume that all topological spaces involved are Hausdorff — unless explicitly stated otherwise.

Definition 1. Let X, Y and Z be topological spaces. A map $f\colon X\times Y\to Z$, $(x,y)\mapsto f(x,y)$, is called

(*i*) *right* [*left*] *continuous* if it is continuous in the right [left] variable; i.e., for every fixed $x_0\in X$ [$y_0\in Y$] the map $Y\to Z$, $y\mapsto f(x_0,y)$ [$X\to Z$, $x\mapsto f(x,y_0)$] is continuous;

(*ii*) *separately continuous* if it is both left and right continuous;

(*iii*) *jointly continuous* if it is continuous as a map between the product space $X\times Y$ and the space Z.

Definition 2. Let S be a non-void topological space which is provided with an associative multiplication (a semigroup operation) $\mu\colon S\times S\to S$, $(x,y)\mapsto\mu(x,y)=xy$. Then the pair (S,μ) is called