International scientific conference

# “Algebraic and Geometric Methods of Analysis”

## Book of abstracts

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## List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
- History and methodology of teaching in mathematics

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# Minimal generating set and structure of wreath product of cyclic groups, comutator of wreath product and the fundamental group of orbit Morse function $\pi_1 O(f)$

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Let $i_j$ be the orders of $C_{i_j}$. In this work the previous result of the author [1] is strengthen also there is considered new class of *wreathcyclic* groups $\Im$ (let $G \in \Im$) which constructed by formula:

$$G = (\mathop{\wr}_{j_0=0}^{n_0} C_{k_{j_0}}) \times (\mathop{\wr}_{j_1=0}^{n_1} C_{k_{j_1}}) \times \ldots \times (\mathop{\wr}_{j_l=0}^{n_l} C_{k_{j_l}}), 1 \leq k_{j_i} < \infty, n_i < \infty.$$

**Theorem 1.** *If orders of cyclic groups* $\mathbb{C}_{n_i}$, $\mathbb{C}_{n_j}$ *is mutually coprime* $i \neq j$ *then the group* $G = C_{i_1} \wr C_{i_2} \wr \ \ldots \ \wr C_{i_m}$ *admits two generators* $\beta_0$, $\beta_1$.

The subtree of $X^*$ induced by the set of vertices $\cup_{i=0}^{k} X^i$ is denoted by $X^{[k]}$.

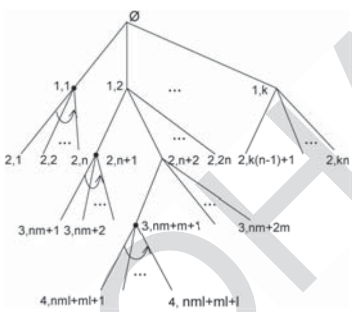


Figure 1.1. Directed automorhism

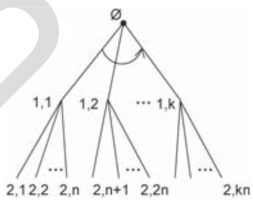


Figure 1.2. Rooted automorhism

We construct the generators of $\mathop{\wr}_{j=0}^{n} C_{i_j}$ as a rooted automorphism $\beta_0$ in Figure 2 and a directed $\beta_1$ along a path $l$ in Figure 1.1 on a rooted labeled truncated tree $X^{[k]}$.

Let $l = x_1 x_2 x_3 ... x_k$ be an finite ray in $X^{[k]}$.

**Definition 2.** We say that the automorphism $g$ of $\mathbb{X}$ is directed along $l$ and we call $l$ the spine of $g$ if all vertex permutations along the ray $l$ and all vertex permutations corresponding to vertices whose distance to the ray $l$ is at least 2 are trivial (Figure 1).

**Definition 3.** An automorphism of $X$ is rooted if all of its vertex permutations that correspond to non-empty words are trivial.

**Corollary 4.** *A center of the group* $\mathbb{Z}\ltimes_{\phi}(\mathbb{Z})^n \simeq (\mathbb{Z}, X)\wr\mathbb{Z}$ *consists of normal closure of diagonal of* $\mathrm{Z}^n$*, trivial an element, and kernel of action by conjugation that is* $n\mathrm{Z}$*. Other words*

$$Z(H) = \langle\, 1;\ \underbrace{h, h, ..., h}_{n}),\ e,\ (n\mathbb{Z}, X)\wr\mathbb{E}\rangle \simeq n\mathbb{Z}\times\mathbb{Z},$$

*where* $h,\ g\in\mathrm{Z}$*,* $Z(H)\simeq n\mathbb{Z}\times\mathbb{Z}$*.*

**Corollary 5.** *A center of a group of form* $\mathbb{Z}\ltimes_{\phi}(\mathcal{B})^n \simeq (\mathbb{Z}, X)\wr\mathcal{B}$ *generates by normal closure of: diagonal of* $\mathcal{B}^n$*, trivial an element, and* $n\mathrm{Z}\underset{X}{\wr}\mathcal{E}$*.*

In our case the Morse function [2] $f$ on $M$ that has the following properties:

1. $f$ is constant on the bound $M$,
2. it has 2 points of maximum at a saddle point,
3. at these 2 points of maximum, the values of the function are equal; in every critical point of $f$ the germ of $f$ is $C^{\infty}$ equivalent to some homogeneous polynomial of 2 real variables without multiple factors.

Consider a group $H$ of automorphisms of $M$ which are induced by the action of diffeomorphisms $h$ of a group $D(M)$ such that preserving the Mebius function $f$, that is, such $h$ are from the stabilizer $S(f)\triangleleft D(M)$. Generators of their stabilizers by right action by diffeomorphisms $\pi_0 S(f|_{X_i}, \partial X_i)$ are $\tau_i$.

The first generator $\rho$ of cyclic group $Z$ realizes shift of Mebius band and second $\tau$ realize rotation of domains $X_i$ of simple connectedness on Mebius band when passing through the twisting point of Mebius band (M).

**Proposition 6.** *The group* $H \simeq \mathbb{Z}\ltimes(\mathbb{Z})^n = \langle\rho,\ \tau\rangle$ *with defined above homomorphism in* $Aut Z^n$ *has two generators and non trivial relations*

$$\rho^n\tau\rho^{-n} = \tau^{-1},\ \rho^i\tau\rho^{-i}\rho^j\tau\rho^{-j} = \rho^j\tau\rho^{-j}\rho^i\tau\rho^{-i},\ 0<i,j<n.$$

*Also this group admits another presentation in generators and relations*

$$\left\langle \rho, \tau_1, ..., \tau_n \,\middle|\, \rho\tau_{i(\bmod n)}\rho^{-1} = \tau_{i+1(\bmod n)},\ \ \tau_i\tau_j = \tau_j\tau_i,\ i,\ j\le n \right\rangle. \tag{1}$$

**Proposition 7.** *The commutator of Sylow 2-subgroup* $(Syl_2 A_{2^k})'$ *has order* $2^{2^k-k-2}$*.*

**Proposition 8.** *The second commutator of Sylow 2-subgroup* $(Syl_2 A_{2^k})$ *has the order* $2^{2^k-3k+1}$*.*

**Corollary 9.** *The Frattini factor of* $(Syl_2 A_{2^k})'$ *is isomorphic to elementary abelian subgroup* $(C_2)^{2k-3}$*. Any minimal generator set of* $(Syl_2 A_{2^k})'$*. has* $2k-3$ *generators.*

**Example 10.** The minimal generating set of $Syl_2'(A_8)$ consists of 3 generators: $(1,3)(2,4)(5,7)(6,8)$, $(1,2)(3,4)$, $(1,3)(2,4)(5,8)(6,7)$. The commutator $Syl_2'(A_8)\simeq C_2^3$ that is an elementary abelian 2-group of order 8. Minimal generating set of $Syl_2'(A_{16})$ consist of 5 (that is $2\cdot 4-3$) generators:
$(1,4,2,3)(5,6)(9,12)(10,11), (1,4)(2,3)(5,8)(6,7), (1,2)(5,6),$
$(1,7,3,5)(2,8,4,6)(9,14,12,16)(10,13,11,15), (1,7)(2,8)(3,6)(4,5)(9,16,10,15)\times$
$\times(11,14,12,13)$.

## References


[1] *R. V. Skuratovskii,* Minimal generating sets for wreath products of cyclic groups, groups of automorphisms of Ribe graph and fundumental groups of some Morse functions orbits. (in Russian) Summer school *Algebra, Topology and Analys*, P. 121–123, 2016.

[2] *S. I. Maksymenko,* Deformations of functions on surfaces by isotopic to the identity diffeomorphisms. 2013, arXiv:1311.3347.