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**Book of abstracts**



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ФІТБ ОНАФТ

# Minimal generating set and structure of wreath product of cyclic groups, comutator of wreath product and the fundamental group of orbit Morse function $\pi_1 O(f)$

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Let  $i_j$  be the orders of  $C_{i_j}$ . In this work the previous result of the author [1] is strengthen also there is considered new class of *wreathcyclic* groups  $\mathfrak{S}$  (let  $G \in \mathfrak{S}$ ) which constructed by formula:

$$G = \left( \prod_{j_0=0}^{n_0} C_{k_{j_0}} \right) \times \left( \prod_{j_1=0}^{n_1} C_{k_{j_1}} \right) \times \dots \times \left( \prod_{j_l=0}^{n_l} C_{k_{j_l}} \right), 1 \leq k_{j_i} < \infty, n_i < \infty.$$

**Theorem 1.** *If orders of cyclic groups  $C_{n_i}$ ,  $C_{n_j}$  is mutually coprime  $i \neq j$  then the group  $G = C_{i_1} \wr C_{i_2} \wr \dots \wr C_{i_m}$  admits two generators  $\beta_0, \beta_1$ .*

The subtree of  $X^*$  induced by the set of vertices  $\cup_{i=0}^k X^i$  is denoted by  $X^{[k]}$ .

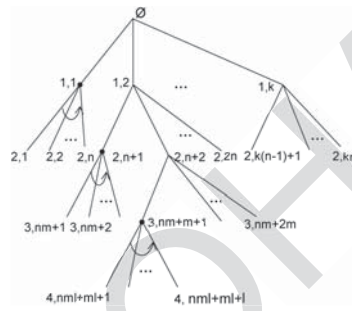


FIGURE 1.1. Directed automorphism

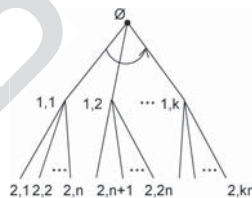


FIGURE 1.2. Rooted automorphism

We construct the generators of  $\prod_{j=0}^n C_{i_j}$  as a rooted automorphism  $\beta_0$  in Figure 2 and a directed  $\beta_1$  along a path  $l$  in Figure 1.1 on a rooted labeled truncated tree  $X^{[k]}$ .

Let  $l = x_1 x_2 x_3 \dots x_k$  be an finite ray in  $X^{[k]}$ .

**Definition 2.** We say that the automorphism  $g$  of  $\mathbb{X}$  is directed along  $l$  and we call  $l$  the spine of  $g$  if all vertex permutations along the ray  $l$  and all vertex permutations corresponding to vertices whose distance to the ray  $l$  is at least 2 are trivial (Figure 1).

**Definition 3.** An automorphism of  $X$  is rooted if all of its vertex permutations that correspond to non-empty words are trivial.

**Corollary 4.** A center of the group  $\mathbb{Z} \times_{\phi} (\mathbb{Z})^n \simeq (\mathbb{Z}, X) \wr \mathbb{Z}$  consists of normal closure of diagonal of  $\mathbb{Z}^n$ , trivial an element, and kernel of action by conjugation that is  $n\mathbb{Z}$ . Other words

$$Z(H) = \langle 1; \underbrace{h, h, \dots, h}_n, e, (n\mathbb{Z}, X) \wr \mathbb{Z} \rangle \simeq n\mathbb{Z} \times \mathbb{Z},$$

where  $h, g \in \mathbb{Z}$ ,  $Z(H) \simeq n\mathbb{Z} \times \mathbb{Z}$ .

**Corollary 5.** A center of a group of form  $\mathbb{Z} \times_{\phi} (\mathcal{B})^n \simeq (\mathbb{Z}, X) \wr \mathcal{B}$  generates by normal closure of: diagonal of  $\mathcal{B}^n$ , trivial an element, and  $n\mathbb{Z} \wr \mathcal{E}$ .

In our case the Morse function [2]  $f$  on  $M$  that has the following properties:

- (1)  $f$  is constant on the bound  $M$ ,
- (2) it has 2 points of maximum at a saddle point,
- (3) at these 2 points of maximum, the values of the function are equal; in every critical point of  $f$  the germ of  $f$  is  $C^\infty$  equivalent to some homogeneous polynomial of 2 real variables without multiple factors.

Consider a group  $H$  of automorphisms of  $M$  which are induced by the action of diffeomorphisms  $h$  of a group  $D(M)$  such that preserving the Mebius function  $f$ , that is, such  $h$  are from the stabilizer  $S(f) \triangleleft D(M)$ . Generators of their stabilizers by right action by diffeomorphisms  $\pi_0 S(f|_{X_i}, \partial X_i)$  are  $\tau_i$ .

The first generator  $\rho$  of cyclic group  $Z$  realizes shift of Mebius band and second  $\tau$  realize rotation of domains  $X_i$  of simple connectedness on Mebius band when passing through the twisting point of Mebius band (M).

**Proposition 6.** The group  $H \simeq \mathbb{Z} \times (\mathbb{Z})^n = \langle \rho, \tau \rangle$  with defined above homomorphism in  $AutZ^n$  has two generators and non trivial relations

$$\rho^n \tau \rho^{-n} = \tau^{-1}, \quad \rho^i \tau \rho^{-i} \rho^j \tau \rho^{-j} = \rho^j \tau \rho^{-j} \rho^i \tau \rho^{-i}, \quad 0 < i, j < n.$$

Also this group admits another presentation in generators and relations

$$\langle \rho, \tau_1, \dots, \tau_n \mid \rho \tau_i (\text{mod } n) \rho^{-1} = \tau_{i+1} (\text{mod } n), \quad \tau_i \tau_j = \tau_j \tau_i, \quad i, j \leq n \rangle. \quad (1)$$

**Proposition 7.** The commutator of Sylow 2-subgroup  $(Syl_2 A_{2^k})'$  has order  $2^{2^k - k - 2}$ .

**Proposition 8.** The second commutator of Sylow 2-subgroup  $(Syl_2 A_{2^k})$  has the order  $2^{2^k - 3k + 1}$ .

**Corollary 9.** The Frattini factor of  $(Syl_2 A_{2^k})'$  is isomorphic to elementary abelian subgroup  $(C_2)^{2^k - 3}$ . Any minimal generator set of  $(Syl_2 A_{2^k})'$  has  $2k - 3$  generators.

**Example 10.** The minimal generating set of  $Syl_2'(A_8)$  consists of 3 generators:  $(1, 3)(2, 4)(5, 7)(6, 8)$ ,  $(1, 2)(3, 4)$ ,  $(1, 3)(2, 4)(5, 8)(6, 7)$ . The commutator  $Syl_2'(A_8) \simeq C_2^3$  that is an elementary abelian 2-group of order 8. Minimal generating set of  $Syl_2'(A_{16})$  consist of 5 (that is  $2 \cdot 4 - 3$ ) generators:  $(1, 4, 2, 3)(5, 6)(9, 12)(10, 11)$ ,  $(1, 4)(2, 3)(5, 8)(6, 7)$ ,  $(1, 2)(5, 6)$ ,  $(1, 7, 3, 5)(2, 8, 4, 6)(9, 14, 12, 16)(10, 13, 11, 15)$ ,  $(1, 7)(2, 8)(3, 6)(4, 5)(9, 16, 10, 15) \times (11, 14, 12, 13)$ .

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