

International
Scientific Conference



Algebraic
and Geometric
Methods
of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Persistent interaction topology in data analysis

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Abstract. Topological data analysis, as a tool for extracting topological features and characterizing geometric shapes, has had tremendous success across diverse fields. Its key mathematical techniques include persistent homology and the recently developed persistent Laplacians. However, classic mathematical models like simplicial complexes often struggle to provide a localized topological description for interactions or individual elements within a complex system involving a specific set of elements. In this work, we introduce persistent interaction homology and persistent interaction Laplacian that emphasize individual interacting elements in the system. We demonstrate the stability of persistent interaction homology as a persistent module. Furthermore, for a finite discrete set of points in the Euclidean space, we provide the construction of persistent interaction Vietoris-Rips complexes and compute their interaction homology and interaction Laplacians. The proposed methods hold significant promise for analyzing heterogeneously interactive data and emphasizing specific elements in data. Their utility for data science is demonstrated with applications to molecules.

Reeb vector field as isometric embedding

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Considering **Sasaki metric** on the unit tangent bundle T_1M , a map $\xi : (M, g) \rightarrow (T_1M, G)$, defining by $\xi(x) = (x, \xi(x))$, is **isometric embedding** only if ξ is **parallel**. The rigidity of Sasaki metric motivates many authors consider various deformations of Sasaki metric (see [1, 5, 6, 7, 11]). In particular, Domenico Perrone [8] studied **Reeb vector fields** with respect to a **Riemannian g-natural metrics** on the unit tangent bundle.

A **Riemannian g-natural metric** [1, 2] on the unit tangent bundle T_1M is defined by

$$G_{(x,\xi)}(X^h, Y^h) = (a + c)g_x(X, Y) + dg_x(X, \xi)g_x(Y, \xi),$$

$$G_{(x,\xi)}(X^h, Y^v) = bg_x(X, Y),$$

$$G_{(x,\xi)}(X^v, Y^v) = ag_x(X, Y),$$

where $a, b, c, d = \text{const}$, $a > 0$.

A **contact metric manifold** is defined by a set (M, g, η, ξ, ϕ) , where M is a differential $2n + 1$ -dimensional manifold, ϕ is a tensor field of type $(1, 1)$, ξ is a vector field, η is 1-form satisfying

$$\eta(\xi) = 1, \quad \phi^2 = -I + \eta \otimes \xi,$$

$$\begin{aligned}g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y), \\(d\eta)(X, Y) &= g(X, \phi Y).\end{aligned}$$

Moreover, ξ is called **Reeb vector field** and η uniquely defines ξ by the conditions

$$\eta(\xi) = 1, \quad d\eta(\xi, X) = 0.$$

The Reeb vector field of a contact manifold plays a fundamental role in the study of the Riemannian geometry of a contact metric manifold (see [3]). A contact metric manifold (M, g, η, ξ, ϕ) is called **K-contact manifold** if ξ is **Killing vector field**. Moreover, if

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X,$$

then the contact metric manifold (M, g, η, ξ, ϕ) is called **Sasakian manifold**.

Domenico Perrone [8] showed that there are **non-parallel** unit vector fields which define **isometric embeddings** with respect to a family of **Riemannian g-natural metrics** on the unit tangent bundle that depend on two parameters, which does not include the Sasaki metric.

Proposition 1. *Let (M, g, η, ξ, ϕ) be a contact metric manifold, $\dim M = 2n + 1$, and let G be a Riemannian g -natural metric on T_1M with $c = 1 - 2a$. Then the map $\xi : (M, g) \rightarrow (T_1M, G)$ is an isometric embedding if and only if $d = a$ and M is K -contact manifold.*

If $\xi(x)$ is a unit vector field on M , then it defines a map $\xi : M \rightarrow T_1M$, defining by $\xi(x) = (x, \xi(x))$. From geometrical viewpoint $\xi(M)$ is explicitly given submanifold in T_1M .

A unit vector field ξ is said to be **harmonic** (see [9]) if it is a critical point of the energy functional defined on the space of all unit vector fields. The corresponding map $\xi : M \rightarrow T_1M$ is said to be **harmonic map** if it is a critical point of the energy functional defined on the space of all maps from M to T_1M . Note that a harmonic vector field ξ does not define, in general, a harmonic map from $\xi : M \rightarrow T_1M$.

Minimal submanifold is a submanifold with vector of mean curvature zero. A unit vector field ξ on Riemannian manifold M is called **minimal** (see [4]) if the image of (local) embedding $\xi : M \rightarrow T_1M$ is minimal submanifold in the unit tangent bundle T_1M . Note that an isometric immersion is minimal if and only if it is a harmonic map.

Domenico Perrone [8] suggested the following theorems.

Theorem 2. *The Reeb vector field ξ of a K -contact manifold (M, g, η, ξ, ϕ) defines a harmonic map $\xi : (M, g) \rightarrow (T_1M, G)$ for any Riemannian g -natural metric G on T_1M .*

Theorem 3. *Let (M, g, η, ξ, ϕ) be a K -contact manifold. Let \mathcal{F} be the family of all Riemannian g -natural metrics on T_1M defined by the parameters*

$$0 < a < 1, \quad b^2 < a(1 - a), \quad c = 1 - 2a, \quad d = a.$$

Then, the Reeb vector field determines a minimal isometric immersion $\xi : (M, g) \rightarrow (T_1M, G)$ for any $G \in \mathcal{F}$.

Totally geodesic submanifold is a submanifold such that all geodesics in the submanifold are also geodesics of the surrounding manifold. A unit vector field ξ on Riemannian manifold M is called **totally geodesic** (see [10]) if the image of (local) embedding $\xi : M \rightarrow T_1M$ is totally geodesic submanifold in the unit tangent bundle T_1M . The corresponding map $\xi : M \rightarrow T_1M$ is said to be **totally geodesic map**. Namely, ξ is total geodesic if the second fundamental form of the map $\xi : M \rightarrow T_1M$ vanishes. Note that every totally geodesic map $\xi : M \rightarrow T_1M$ is harmonic and minimal.

The concept of totally geodesicity arises naturally in connection with the concepts of harmonicity and minimality. As a result, we have the following theorem.

Theorem 4. *Let (M, g, η, ξ, ϕ) be a K -contact metric manifold, $\dim M = 2n + 1$, and let G be a Riemannian g -natural metric on T_1M with $c = 1 - 2a$ and $d = a$. Then the Reeb vector field ξ defining the isometric embedding $\xi : (M, g) \rightarrow (T_1M, G)$ is totally geodesic if and only if M is Sasakian manifold.*

Thus totally geodesic property of the Reeb vector fields as isometric embeddings is distinguished Sasakian manifold among K -contact metric manifold with the Riemannian g -natural metrics on the unit tangent bundle.

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Classification of smooth structures on line with two origins

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We classified differentiable structures on a line \mathbf{L} with two origins begin a non-Hausdorff but T_1 one-dimensional manifold obtained by "doubling" 0.

Definition 1. Let τ be the standard topology on \mathbb{R} . Then $\mathbf{L} = \mathbb{R} \sqcup \bar{0}$ is a disjoint union of \mathbb{R} with some point $\bar{0}$ endowed with the following topology:

$$\eta = \tau \cup \{(W \setminus 0) \cup \{\bar{0}\} : 0 \in W \in \tau\}$$

whose elements are elements of τ and also open neighborhoods of 0 in which 0 is replaced with $\bar{0}$.

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