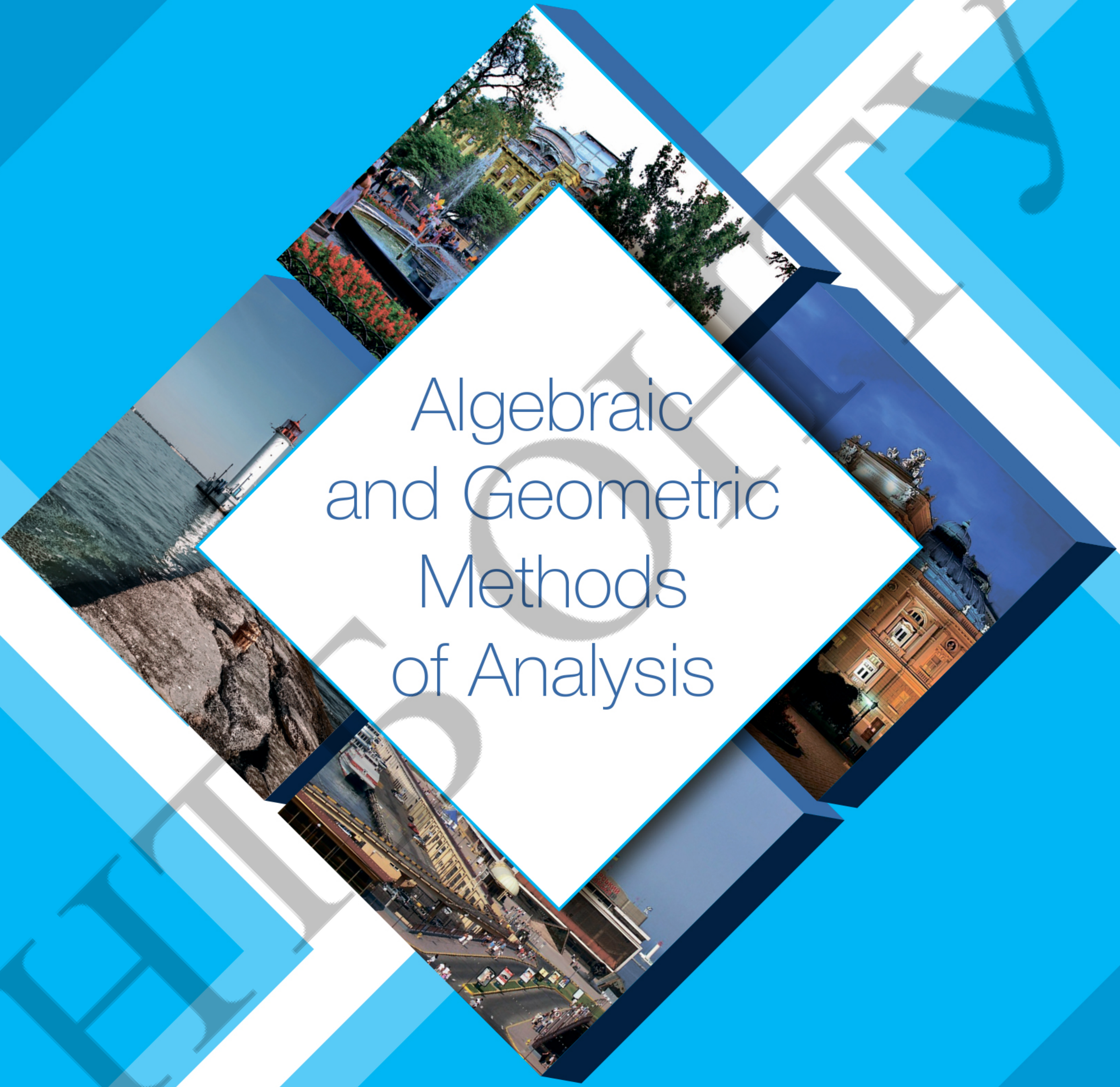
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Vladimir Balan** (*Bucharest, Romania*)
- **Taras Banakh** (*Lviv, Ukraine*)
- **Dmytro Bolotov** (*Kharkiv, Ukraine*)
- **Vyacheslav Boyko** (*Kyiv, Ukraine*)
- **Yulia Fedchenko** (*Odesa, Ukraine*)
- **Oleg Gutik** (*Lviv, Ukraine*)
- **Olena Karlova** (*Chernivtsi, Ukraine*)
- **Volodymyr Kiosak** (*Odesa, Ukraine*)
- **Nadiia Konovenko** (*Odesa, Ukraine*)
- **Volodymyr Lyubashenko** (*Kyiv, Ukraine*)
- **Sergiy Maksymenko** (*Kyiv, Ukraine*)
- **Koji Matsumoto** (*Yamagata, Japan*)
- **Piotr Mormul** (*Warsaw, Poland*)
- **Maryna Nesterenko** (*Kyiv, Ukraine*)
- **Roman Popovych** (*Kyiv, Ukraine*)
- **Alexandr Prishlyak** (*Kyiv, Ukraine*)
- **Aleksandr Savchenko** (*Kherson, Ukraine*)

Organizing commitee

- **Nadiia Konovenko** (*Odesa, Ukraine*)
- **Yuliya Fedchenko** (*Odesa, Ukraine*)
- **Mykola Lysynskiy** (*Kyiv, Ukraine*)
- **Bohdan Mazhar** (*Kyiv, Ukraine*)
- **Sergiy Maksymenko** (*Kyiv, Ukraine*)
- **Alexandr Prishlyak** (*Kyiv, Ukraine*)

Definition 2. A metric space (X, d) is called a *weak Rolewicz space*, if there exist $C > 0$, a sequence $(\varepsilon_n)_{n=1}^{\infty}$ of positive reals which tends to zero and a sequence $(R_n)_{n=1}^{\infty}$ of functions $R_n : X \times X \to X$ such that for all $x, y \in X$

(1) if $d(x, y) \le \varepsilon_n$, then $R_n(x, y) = x$,
(2) $d(R_n(x, y), y) \le C \cdot \varepsilon_n$ for $n = 1, 2, \dots$.

Every convex subset of a metric vector space is an example of a weak Rolewicz space. Moreover, there are zero dimensional examples of Rolewicz spaces.

Proposition 3. *If Y is a weak Rolewicz space, then a uniform limit $f : X \to Y$ of a sequence of Baire 1 functions $f_n : X \to Y$ belongs to the first Baire class.*

The next theorem is the main result of the talk.

Theorem 4. *Let X, Y be metric spaces and X is locally compact weak Rolewicz space. Then for every continuous bijection $f : X \to Y$ the inverse mapping $f^{-1} : Y \to X$ is Baire 1.*

References

[1] Stefan Rolewicz. On inversion of non-linear transformations, *Studia Mathematica*, 17 : 79–83, 1958.
[2] Olena Karlova, Mykhaylo Lukan, R-spaces and uniform limit of sequences of the first Baire class, *Bukovinian Mathematical Journal*, 7 (2) : 39–47, 2019.

On boundary controllability problems for the heat equation with variable coefficients on a half-axis

Larissa Fardigola
(B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Nauky Ave., Kharkiv, 61103, Ukraine;
V.N. Karazin Kharkiv National University, 4 Svobody Sq., Kharkiv, 61022, Ukraine)
E-mail: `fardigola@ilt.kharkov.ua`

Kateryna Khalina
(B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Nauky Ave., Kharkiv, 61103, Ukraine)
E-mail: `khalina@ilt.kharkov.ua`

Consider the control system for the heat equation on a half-axis:

$$w_t = \frac{1}{\rho}\left(k w_x\right)_x + \gamma w, \qquad x \in \mathbb{R}_+,\ t \in (0, T), \tag{1}$$

$$\left(\sqrt{k/\rho}\, w_x\right)\Big|_{x=0} = u, \qquad t \in (0, T), \tag{2}$$

$$w(\cdot, 0) = w^0, \qquad x \in \mathbb{R}_+, \tag{3}$$

where $\mathbb{R}_+ = (0, +\infty)$; T is a positive constant; ρ, k, γ, w^0 are given functions; $u \in L^\infty(0, T)$ is a control. We assume ρ and k are positive on $[0, +\infty)$, $\rho, k \in C^1[0, +\infty)$, $(\rho k) \in C^2[0, +\infty)$, $(\rho k)'(0) = 0$, and

$$\sigma(x) = \int_0^x \sqrt{\rho(\mu)/k(\mu)}\, d\mu \to +\infty \qquad \text{as } x \to +\infty.$$

Moreover, we assume

$$(Q_2(\rho, k) - \gamma) \in L^\infty(0, +\infty) \bigcap C^1[0, +\infty) \qquad \text{and} \qquad \sigma\sqrt{\rho/k}\,(Q_2(\rho, k) - \gamma) \in L^1(0, +\infty),$$

where $Q_2(\rho,k)=\sqrt{k/\rho}\big(Q_1(\rho,k)\big)'+\big(Q_1(\rho,k)\big)^2$, $Q_1(\rho,k)=\sqrt{k/\rho}(k\rho)'/(4k\rho)$.

Control system (1)–(3) is considered in modified Sobolev spaces. Let $\varphi\in L^2_{\text{loc}}(\mathbb{R}_+)$. We define the modified derivative $\mathbb{D}_{\rho k}$ by the rule

$$\mathbb{D}_{\rho k}\varphi=\sqrt{k/\rho}\varphi'+Q_1(\rho,k)\varphi.$$

If, in addition, $\mathbb{D}_{\rho k}\varphi\in L^2_{\text{loc}}(\mathbb{R}_+)$ and $(\mathbb{D}_{\rho k}\varphi)'\in L^2_{\text{loc}}(\mathbb{R}_+)$ we can consider $\mathbb{D}^2_{\rho k}\varphi$:

$$\mathbb{D}^2_{\rho k}\varphi=\frac{1}{\rho}\big(k\varphi'\big)'+Q_2(\rho,k)\varphi.$$

Obviously, $\mathbb{D}^m_{\rho k}\varphi=\varphi^{(m)}$ if $\rho=k=1$, $m=0,1$.

Denote

$$L^2_\rho(\mathbb{R}_+)=\{f\in L^2_{\text{loc}}(\mathbb{R}_+)\mid\sqrt{\rho}f\in L^2(\mathbb{R}_+)\}$$

with the norm $\|f\|_{L^2_\rho(\mathbb{R}_+)}=\|\sqrt{\rho}f\|_{L^2(\mathbb{R}_+)}$, $f\in L^2_\rho(\mathbb{R}_+)$.

Now consider the modified Sobolev spaces

$$\overset{\circ}{\mathbb{H}}{}^0=L^2_\rho(\mathbb{R}_+),\quad \overset{\circ}{\mathbb{H}}{}^1=\{\varphi\in L^2_\rho(\mathbb{R}_+)\mid\mathbb{D}_{\rho k}\varphi\in L^2_\rho(\mathbb{R}_+)\text{ and }\varphi(0^+)=0\}$$

with the norm

$$[\![\varphi]\!]^p=\left(\sum_{m=0}^{p}\binom{p}{m}\left(\|\mathbb{D}^m_{\rho k}\varphi\|_{L^2_\rho(\mathbb{R}_+)}\right)^2\right)^{1/2},\quad\varphi\in\overset{\circ}{\mathbb{H}}{}^p,\quad p=0,1,$$

and the dual space $\overset{\circ}{\mathbb{H}}{}^{-p}=\left(\overset{\circ}{\mathbb{H}}{}^p\right)^*$, $p=0,1$, with the norm associated with the strong topology of the adjoint space.

In control system (1)–(3), we suppose $\left(\frac{d}{dt}\right)^p w:[0,T]\to\overset{\circ}{\mathbb{H}}{}^{1-2p}$, $p=0,1$; $w^0\in\overset{\circ}{\mathbb{H}}{}^1$.

Let $T>0$, $w^0\in\overset{\circ}{\mathbb{H}}{}^1$. By $\mathcal{R}_T(w^0)$, denote the set of all states $w^T\in\overset{\circ}{\mathbb{H}}{}^1$ for which there exists a control $u\in L^\infty(0,T)$ such that there exists a unique solution w to (1)–(3) and $w(\cdot,T)=w^T$.

Definition 1. A state $w^0\in\overset{\circ}{\mathbb{H}}{}^1$ is said to be *null-controllable* with respect to system (1)–(3) in a given time $T>0$ if $0\in\mathcal{R}_T(w^0)$.

Definition 2. A state $w^0\in\overset{\circ}{\mathbb{H}}{}^1$ is said to be *approximately controllable* to a state $w^T\in\overset{\circ}{\mathbb{H}}{}^1$ with respect to system (1)–(3) in a given time $T>0$ if $w^T\in\overline{\mathcal{R}_T(w^0)}$, where the closure is considered in the space $\overset{\circ}{\mathbb{H}}{}^1$.

Consider also the control system with the simplest heat operator (the case $\rho=k=1$, $\gamma=0$):

$$z_t=z_{yy},\qquad y\in\mathbb{R}_+,\ t\in(0,T),\tag{4}$$

$$z_y(0,\cdot)=v,\qquad t\in(0,T),\tag{5}$$

$$z(\cdot,0)=z^0,\qquad y\in\mathbb{R}_+,\tag{6}$$

where $v\in L^\infty(0,T)$ is a control, $\left(\frac{d}{dt}\right)^m z:[0,T]\to H^{1-2m}$, $m=0,1$, $z^0\in H^1$. Here H^p, $p=-1,0,1$, are the Sobolev spaces.

Controllability problems for system (4)–(6) were investigated in [1].

To study controllability problems for system (1)–(3), we use the transformation operator $\widehat{\mathbb{T}}:H^{-1}\to\overset{\circ}{\mathbb{H}}{}^{-1}$. It was introduced and studied in [2]. In particular, it has been proved therein that $\widehat{\mathbb{T}}$ is a continuous one-to-one mapping between the spaces H^p and $\overset{\circ}{\mathbb{H}}{}^p$, $p=-1,0,1$.

In the present talk, we prove that the transformation operator $\widehat{\mathbb{T}}$ is one-to-one mapping between the sets of the solutions to system (4)–(6) and to system (1)–(3). The application of the operator $\widehat{\mathbb{T}}$ allows us to conclude that the control system (1)–(3) replicates the controllability properties of the control system (4)–(6) and vice versa. A relation between controls u and v is also found. Thus, using obtained results for control system (4)–(6), we obtain the following main results for control system (1)–(3).

Theorem 3. *If a state* $w^0 \in \mathring{\mathbb{H}}^1$ *is null-controllable with respect to system* (1)–(3) *in a time* $T > 0$*, then* $w^0 = 0$*.*

Theorem 4. *Each state* $w^0 \in \mathring{\mathbb{H}}^1$ *is approximately controllable to any target state* $w^T \in \mathring{\mathbb{H}}^1$ *with respect to system* (1)–(3) *in a given time* $T > 0$*.*

All obtained results have been published in [3].

References

[1] Larissa Fardigola and Kateryna Khalina. Controllability problems for the heat equation on a half-axis with a bounded control in the Neumann boundary condition. *Math. Control Relat. Fields*, 11(1) : 211–236, 2021.

[2] L.V. Fardigola. Transformation operators and modified Sobolev spaces in controllability problems on a half-axis. *J. Math. Phys. Anal. Geom.*, 12(1) : 17–47, 2016.

[3] Larissa Fardigola and Kateryna Khalina. Controllability problems for the heat equation with variable coefficients on a half-axis controlled by the Neumann boundary condition. *J. Math. Phys. Anal. Geom.*, 19(3) : 616–641, 2023.

On explicit reconstruction of real algebraic maps locally like moment maps

Naoki Kitazawa
(Institute of Mathematics for Industry, Kyushu University)
E-mail: `n-kitazawa@imi.kyushu-u.ac.jp / naokikitazawa.formath@gmail.com`

Real algebraic geometry studies real algebraic varieties. Nash and Tognoli's theory shows that a smooth closed manifold is realized as the zero set of a real algebraic map and non-singular. It is also well-known that smooth maps between non-singular real algebraic manifolds are approximated by real algebraic maps in general. For related history and terminologies and notions, see [5] for example.

Our study is on explicit construction of real algebraic sets and maps. The k-dimensional unit sphere S^k in the $(k+1)$-dimensional real affine space $\mathbb{R}^{k+1}$ and its canonical embedding into higher dimensional real affine spaces (and the compositions with canonical projections) give simplest examples. In general, it is very difficult to give real algebraic sets and important real algebraic maps on them very explicitly. Here, we reconstruct real algebraic maps generalizing the canonical projections of the unit spheres $S^k \subset \mathbb{R}^{k+1}$ from given regions in the target spaces. We discuss this from the viewpoint of global singularity theory and differential topology of manifolds.

Theorem 1. [2] *Let* l_1, l_2 *and* n *be positive integers. Let* $D \subset \mathbb{R}^n$ *be an open subset. Let* $\{S_j\}_{j=1}^{l_1}$ *be a family of non-singular real algebraic hypersurfaces of* $\mathbb{R}^n$*. Let* S_j *be also the zero set of a real polynomial* f_j*. We also assume the following.*