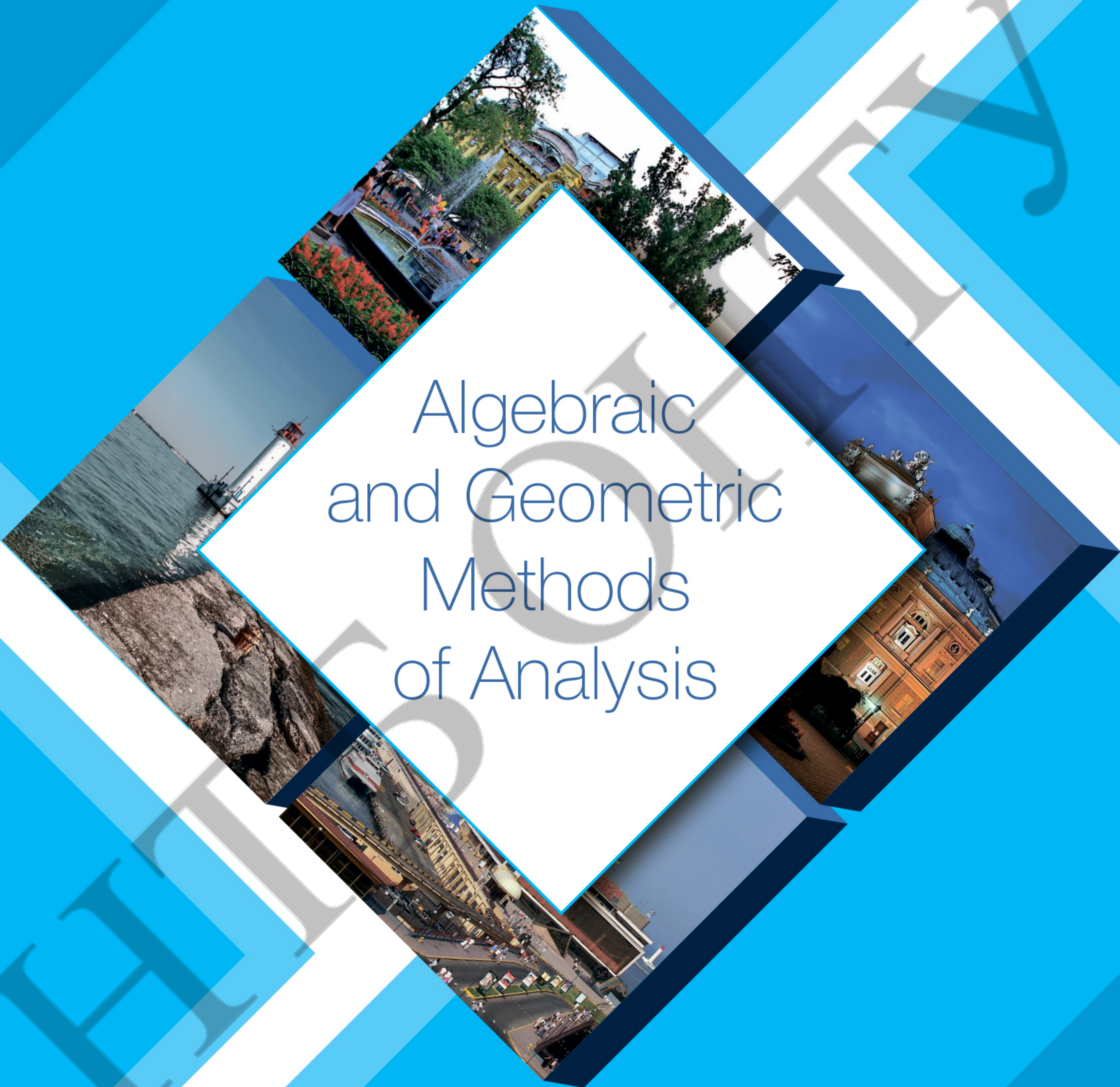
International Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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neat perturbation of rank-two Goursat distributions in the underlying dimension 7 we obtain now a 1-parameter family of strongly nilpotent pairwise nonequivalent distribution germs. That family is given in local coordinates that happen to be already adapted. The members of the family are all quasi-homogeneous with respect to the weights defined by the small growth vector, one and the same for all members of the family. This property automatically yields their strong nilpotency, and also facilitates a proof of their pairwise non-equivalence.

References

[1] Piotr Mormul. Do moduli of Goursat distributions appear on the level of nilpotent approximations? *Trends in Mathematics*. Birkhäuser, 2006, 229–246.

[2] Piotr Mormul. A single nilpotent approximation for a family of nonequivalent distributions. *J. Math. Sci. Adv. Appl.* **26** (2014), 1–27.

Extending of partial metrics

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A function $p : X^2 \to [0, +\infty)$ is called *a partial metric* on X if for every $x, y, z \in X$ the following conditions

(p_1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$;
(p_2) $p(x, x) \le p(x, y)$;
(p_3) $p(x, y) = p(y, x)$;
(p_4) $p(x, z) \le p(x, y) + p(y, z) - p(y, y)$.

are true.

For any partial metric $p : X^2 \to [0, +\infty)$ the function $q_p : X^2 \to [0, +\infty)$, $q_p(x, y) = p(x, y) - p(x, x)$, is a quasi-metric on X and the topology of the partial metric space (X, p) is the topology τ_q of the quasi-metric space (X, q_p). Moreover, the function $d_p : X^2 \to [0, +\infty)$, $d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ is a metric on X.

The following theorem was proved by F. Hausdorff in 1930.

Theorem 1. *Let X be a metrizable space, $A \subseteq X$ be a closed subset and $d_A : A^2 \to \mathbb{R}$ be a compatible metric on A. Then there exists a compatible metric $d : X^2 \to \mathbb{R}$ on X such that $d|_{A^2} = d_A$.*

Problem 2. Let X be a partial metrizable space, $A \subseteq X$ be a closed subset and $p_A : A^2 \to \mathbb{R}$ be a compatible partial metric on A. Does there exist a compatible partial metric $p : X^2 \to \mathbb{R}$ on X such that $p|_{A^2} = p_A$?

Proposition 3. *Let (X, p) be a partial metric space. Then the function $f : X \to \mathbb{R}$, $f(x) = p(x, x)$, is an 1-Lipschitz function with respect to the metric d_p.*

Proposition 4. *Let (X, d) be a metric space and $f : X \to [0, +\infty)$ be an 1-Lipschitz function. Then the function $p : X^2 \to \mathbb{R}$,*

$$p(x, y) = \frac{1}{2}(d(x, y) + f(x) + f(y)),$$

is a partial metric on X such that $d = d_p$ and $p(x, x) = f(x)$ for every $x \in X$.

Theorem 5. *Let X be a quasi-pseudometrizable space, A be a closed subset of X and $q_A : A^2 \to \mathbb{R}$ ba a compatible bounded quasi-pseudometric on A. Then there exists a compatible quasi-pseudometric q on X such that $q|_{A^2} = q_A$.*

Corollary 6. *Let X be a partial metrizable space, A be a closed subset of X and $p_A : A^2 \to \mathbb{R}$ ba a compatible partial metric on A such that q_{p_A} is bounded. Then there exists a compatible partial metric p on X such that $p|_{A^2} = p_A$.*

Proposition 7. *There exist a quasi-pseudometric space (X, q), a τ_q-closed set $A \subseteq X$ and a quasi-pseudometric p on A such that*

(1) q and p are equivalent on A;

(2) $\tau_r \not\subseteq \tau_q$ for every extension r of p on X.

References

[1] Hausdorff F. *Erweiterung einer stetigen Abbildung*, Fund. Math., **16** (1930), 353-360.

[2] Hušek M. *Extension mappings and pseudometrics*, Extracta Mathematicae, **25** (3) (2010), 277-308.

[3] Matthews S.G. *Partial Metric Topology*, Proc. 8th Summer Conference on General Topology and Applications, Ann. New York Acad. Sci. **728** (1994), 183-197.

Axiomatic Development of Complexity Theory for Finite Groups

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"In any field of mathematics, the study of complexity is the first level of sophistication beyond knowing the building blocks." – John Rhodes

What are the simplest ways to construct a finite group from its atomic constituents? To understand part-whole relations between finite simple groups ('atoms') and the global structure of finite groups, we axiomatize complexity measures on finite groups. From the Jordan-Hölder theorem and Frobenius-Kalužnin-Krasner-Lagrange embedding in an iterated wreath product, any finite group G can be constructed from a unique collection of simple groups, its Jordan-Hölder factors, each with well-defined multiplicities through iterated extension by simple groups. What is the least number of levels needed in such a hierarchical construction if a level is allowed to include several of these atomic pieces? To pose and answer this question rigorously, we give a natural set of hierarchical complexity axioms for finite groups relating to constructability, extension, quotients, and products, and prove these axioms are satisfied by a unique maximal complexity function **cx**. We prove this function is the same as the minimal number of "spans of gems" or "completely reducible groups" (i.e., direct products of simple groups) in a subnormal series with all factors of this type. Hierarchical complexity **cx** is thus effectively computable, and bounded below by all other complexity measures satisfying the axioms, including generalizations of derived length, Fitting height and solvability. Also, the hierarchical complexity of a normal subgroup is bounded above by the complexity of the whole group, although this is not assumed in the axioms and does not follow from the axioms for general (non-maximal) complexity functions satisfying the axioms.

For solvable finite groups, the unique maximal group complexity measure satisfying the axioms on this class agrees with the restriction of the one for all finite groups, and in addition satisfies an embedding axiom - which decidedly cannot be applied in the general case of all finite groups. In