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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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A generalized Lie-algebraic approach to constructing of integrable fractional dynamical systems

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In the paper [1] a new metrized Lie algebra of fractional integral-differential operators has been introduced and the infinite hierarchy of Lax type Hamiltonian flows on its dual space, which is reduced to the hierarchies of the Lax integrable fractional-differential dynamical systems on coadjoint orbits, has been constructed by use of the Adler-Kostant-Symes Lie-algebraic scheme. In our report we propose a generalization of the described in [1] Lie-algebraic approach to constructing of Lax integrable fractional-differential dynamical systems, which is based on the central extension by the Maurer-Cartan 2-cocycle of the mentioned above operator Lie algebra. By means of this generalized approach we obtain the Lax integrable fractional-differential Kadomtsev-Petviashvily hierarchy, whose quasi-classical approximation leads to the Benney type hydrodynamic systems.

Let us consider the Lie algebra $\mathbb{A}_\alpha := \mathbb{A}_0\{D^\alpha, D^{-\alpha}\}$ (see [1]), which consists of the fractional integral-differential operators in the forms:

$$a_\alpha := \sum_{j \in \mathbb{Z}_+} a_j D^{\alpha(m_\alpha - j)},$$

where $\mathbb{A}_0 := A\{D, D^{-1}\}$ is the Lie algebra of integral-differential operators, $A := W_2^\infty(\mathbb{R}; \mathbb{C}) \cap W_\infty^\infty(\mathbb{R}; \mathbb{C})$, $D^\alpha : A \rightarrow A$ is a Riemann-Liouville fractional derivative, $\alpha \in \mathbb{C} \setminus \mathbb{Z}$, $\text{Re } \alpha \neq 0$, $m_\alpha \in \mathbb{Z}_+$ and $a_j \in \mathbb{A}_0$, $j \in \mathbb{Z}_+$, and possesses the standard commutator $[\cdot, \cdot]$ and invariant with respect to this commutator scalar product:

$$(a_\alpha, b_\alpha) := \int_{\mathbb{R}} \text{res}_D (\text{res}_{D^\alpha} (a_\alpha \circ b_\alpha D^{-\alpha})) dx,$$

where res_{D^α} denotes a coefficient at $D^{-\alpha}$ for any fractional integral-differential operator as well as res_D denotes a coefficient at D^{-1} for any integral-differential operator. The Lie algebra \mathbb{A}_α allows the splitting into the direct sum of its two Lie subalgebras $\mathbb{A}_\alpha = \mathbb{A}_{\alpha,+} \oplus \mathbb{A}_{\alpha,-}$, where $\mathbb{A}_{\alpha,+}$ is the Lie subalgebra of the formal power series by the operator D^α .

One parameterizes the Lie algebra \mathbb{A}_0 by the variable $y \in \mathbb{S}^1$ and constructs the central extension $\hat{\mathbb{A}}_\alpha := \bar{\mathbb{A}}_\alpha \oplus \mathbb{C}$ of the Lie algebra $\bar{\mathbb{A}}_\alpha := \prod_{y \in \mathbb{S}^1} \mathbb{A}_\alpha$ by the Maurer-Cartan 2-cocycle $\omega_2(\cdot, \cdot)$ on $\bar{\mathbb{A}}_\alpha$ with the commutator:

$$\begin{aligned} [(a_\alpha, d), (b_\alpha, e)] &= ([a_\alpha, b_\alpha], \omega_2(a_\alpha, b_\alpha)), \quad (a_\alpha, d), (b_\alpha, e) \in \hat{\mathcal{L}}_\alpha, \\ [a_\alpha, b_\alpha] &= a_\alpha \circ b_\alpha - b_\alpha \circ a_\alpha, \quad \omega_2(a_\alpha, b_\alpha) := \int_{\mathbb{S}^1} (a_\alpha, \partial b_\alpha / \partial y) dy. \end{aligned} \quad (1)$$

The invariant with respect to the commutator (1) scalar product on $\hat{\mathbb{A}}_\alpha$ is given by the relationship:

$$((a_\alpha, d), (b_\alpha, e)) = \int_{\mathbb{S}^1} (a_\alpha, b_\alpha) dy + ed.$$

The Lie-Poisson bracket, deformed by the space endomorphism $\mathcal{R} = (P_+ - P_-)/2 : \bar{\mathbb{A}}_\alpha \rightarrow \bar{\mathbb{A}}_\alpha$, takes the form:

$$\{\gamma, \mu\}_{\mathcal{R}}(\tilde{l}_\alpha) = (\tilde{l}_\alpha, [\mathcal{R}\nabla\gamma(\tilde{l}_\alpha), \nabla\mu(\tilde{l}_\alpha)] + [\nabla\gamma(\tilde{l}_\alpha), \mathcal{R}\nabla\mu(\tilde{l}_\alpha)]) +$$

$$+c\omega_2(\mathcal{R}\nabla\gamma(\tilde{l}_\alpha), \nabla\mu(\tilde{l}_\alpha)) + c\omega_2(\nabla\gamma(\tilde{l}_\alpha), \mathcal{R}\nabla\mu(\tilde{l}_\alpha)),$$

where $\gamma, \mu \in \mathcal{D}(\bar{\mathbb{A}}_\alpha^*)$ are smooth by Frechet functionals on $\bar{\mathbb{A}}_\alpha^* \simeq \bar{\mathbb{A}}_\alpha$, $\tilde{l}_\alpha \in \bar{\mathbb{A}}_\alpha^*$, $c \in \mathbb{C}$, P_\pm being projectors on $\mathbb{A}_{\alpha, \pm}$, and generate the infinite hierarchy of Lax type Hamiltonian flows:

$$\partial\tilde{l}_\alpha/\partial t_j = [(\nabla\gamma_j(\tilde{l}_\alpha))_+, \tilde{l}_\alpha - c\partial/\partial y], \quad (\tilde{l}_\alpha, c) \in \hat{\mathcal{L}}_\alpha^*, \quad j \in \mathbb{N}, \quad (2)$$

by means of the Casimir invariants $\gamma_j \in I(\bar{\mathbb{A}}_\alpha^*)$, $j \in \mathbb{N}$, as Hamiltonians. The Casimir invariants satisfy the relationship:

$$[\tilde{l}_\alpha - c\partial/\partial y, \nabla\gamma_j(\tilde{l}_\alpha)] = 0,$$

and can be found in the forms:

$$\gamma_j(\tilde{l}_\alpha) = \int_{y \in \mathbb{S}^1} Tr(\tilde{l}_\alpha^0 D^{j\alpha}) dy, \quad j \in \mathbb{N}.$$

Here the coefficients of the operator $\tilde{l}_\alpha^0 := D^{j\alpha} + \sum_{k \leq j-1} \hat{C}_k D^{k\alpha}$, $k \in \mathbb{Z}$, are such that $d\hat{C}_k/dx = 0 = d\hat{C}_k/dt_j$, $j \in \mathbb{N}$, and obey the equality:

$$(\tilde{l}_\alpha - c\partial/\partial y) \circ \Phi = \Phi \circ (\tilde{l}_\alpha^0 - c\partial/\partial y), \quad \Phi = 1 + \sum_{r \in \mathbb{N}} \Phi_r D^{-r\alpha}.$$

As an example, one studies the reduction of the hierarchy (2) on the coadjoint orbit related with the element

$$\tilde{l}_\alpha = D^{2\alpha} + D^\alpha \hat{v} + \hat{v} D^\alpha + \hat{u} \in \bar{\mathbb{A}}_\alpha^*,$$

where $\hat{u}, \hat{v} \in \bar{\mathbb{A}}_0$, when $c = 1$. Looking for the gradients of the Casimir invariants in the forms $\nabla\gamma_j(\tilde{l}_\alpha) = D^{m\alpha} + \sum_{k \leq j-1} \hat{a}_{k,j} D^{k\alpha}$, $m \in \mathbb{Z}_+$, $k \in \mathbb{Z}$, one obtains the hierarchy of fractional-differential dynamical systems such as

$$\begin{aligned} d\hat{u}/dt_1 &= \hat{u}_y + [\hat{v}, \hat{u}], & d\hat{u}/dt_2 &= \hat{u}_y, \\ d(\hat{v} + D^\alpha \hat{v} D^{-\alpha})/dt_1 &= [D^\alpha, \hat{u} - \hat{v}^2] D^{-\alpha}, & d(\hat{v} + D^\alpha \hat{v} D^{-\alpha})/dt_2 &= \hat{v}_y, \\ \\ d\hat{u}/dt_3 &= \hat{f}_y + [\hat{f}, \hat{u}] \\ d(\hat{v} + D^\alpha \hat{v} D^{-\alpha})/dt_3 &= \hat{q}_y + ([\hat{q} D^\alpha, \hat{u}] + [D^\alpha \hat{u} + \hat{v} D^\alpha \hat{v}, \hat{u}] + [\hat{f}, \hat{v} D^\alpha + D^\alpha \hat{v}]) D^{-\alpha}, \\ [\hat{q}, D^{2\alpha}] &= D^\alpha [D^\alpha, \hat{u} - \hat{v}^2], \\ [\hat{f}, D^{2\alpha}] &= -(\hat{v}_y D^{2\alpha} + D^\alpha \hat{v}_y D^\alpha + D^{2\alpha} \hat{v}_y) - [\hat{v} D^{2\alpha} + D^{2\alpha} \hat{v}, \hat{u}] - [D^\alpha \hat{u}, \hat{v} D^\alpha + D^\alpha \hat{v}] - \\ &\quad - [\hat{u} D^\alpha, D^\alpha \hat{v}] - [\hat{v}, D^\alpha \hat{v}^2 D^\alpha] - [\hat{q} D^\alpha, \hat{v} D^\alpha + D^\alpha \hat{v}], \\ &\dots \end{aligned} \quad (3)$$

The gradients of corresponding Casimir invariants are written as

$$\begin{aligned} \nabla\gamma_1(\tilde{l}_\alpha) &= D^\alpha + \hat{v} + \sum_{k \leq 0} \hat{a}_{k,1} D^{k\alpha}, \\ \nabla\gamma_2(\tilde{l}_\alpha) &= D^{2\alpha} + (\hat{v} + D^\alpha \hat{v} D^{-\alpha}) D^\alpha + \hat{u} + \sum_{k \leq 0} \hat{a}_{k,2} D^{k\alpha}, \\ \nabla\gamma_3(\tilde{l}_\alpha) &= D^{3\alpha} + (\hat{v} + D^\alpha \hat{v} D^{-\alpha} + D^{2\alpha} \hat{v} D^{-2\alpha}) D^{2\alpha} + \hat{b} D^\alpha + \hat{f} + \sum_{k \leq 0} \hat{a}_{k,3} D^{k\alpha}, \quad \dots, \end{aligned}$$

where $\hat{b} = \hat{u} + D^\alpha \hat{u} D^{-\alpha} + \hat{v} D^\alpha \hat{v} D^{-\alpha} + \hat{q}$. The third system in the hierarchy (3) can be considered as a fractional-differential analog of the Kadomtsev-Petviashvily equation.

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