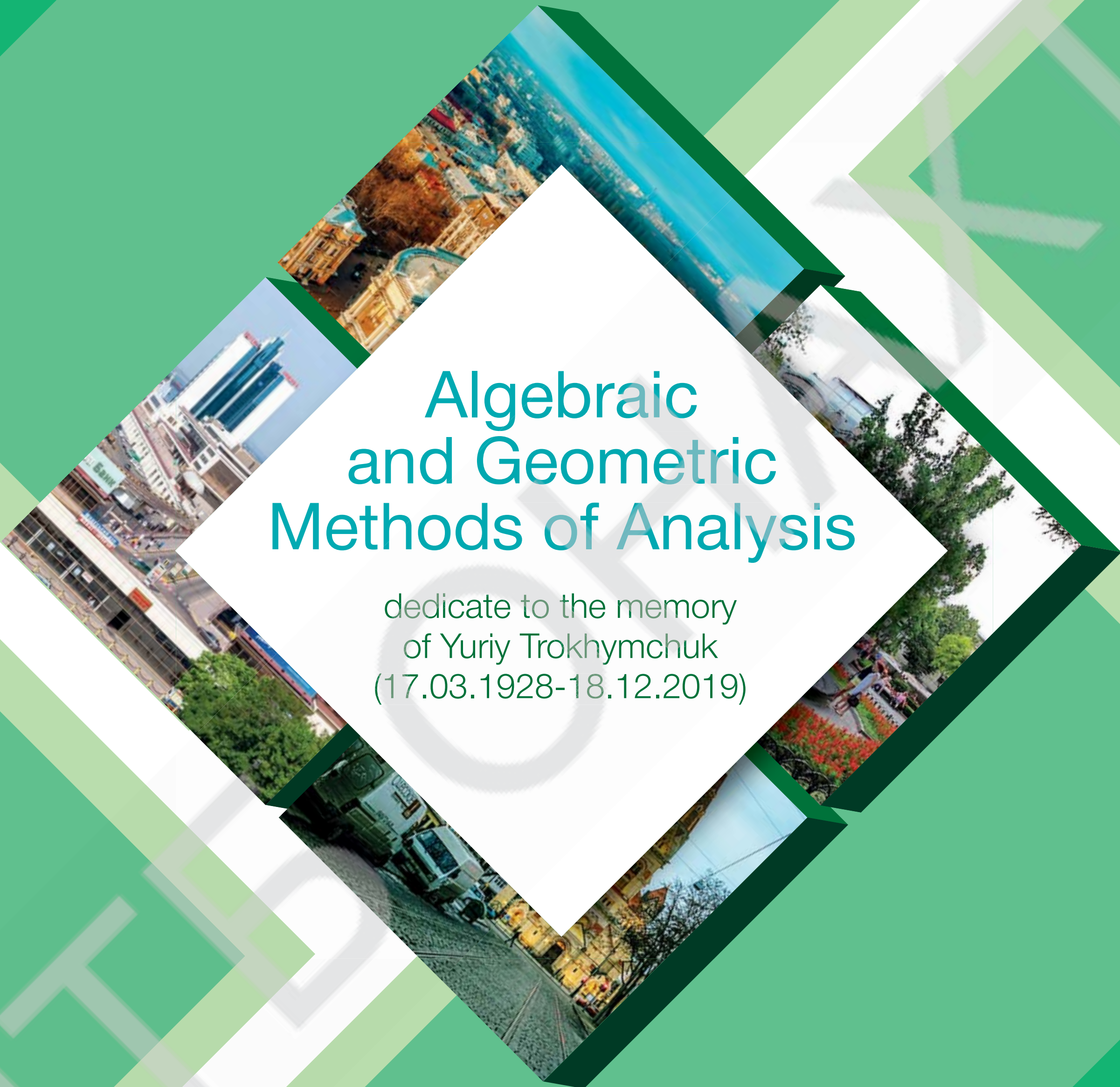
International
Online Conference



Algebraic and Geometric Methods of Analysis

dedicate to the memory
of Yuriy Trokhymchuk
(17.03.1928-18.12.2019)

May 25-28, 2021
Odesa, Ukraine

List of topics

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

Scientific Committee

Drozd Yu.
(*Kyiv, Ukraine*)
Maksymenko S.
(*Kyiv, Ukraine*)
Plaksa S.
(*Kyiv, Ukraine*)
Prishlyak A.
(*Kyiv, Ukraine*)

Bakhtin O.
(*Kyiv, Ukraine*)
Balan V.
(*Bucharest, Romania*)
Banakh T.
(*Lviv, Ukraine*)
Borysenko O.
(*Kharkiv, Ukraine*)
Cherevko Ye.
(*Odesa, Ukraine*)
Fedchenko Yu.
(*Odesa, Ukraine*)
Karlova O.
(*Chernivtsi, Ukraine*)
Kiosak V.
(*Odessa, Ukraine*)
Konovenko N.
(*Odessa, Ukraine*)
Lyubashenko V.
(*Kyiv, Ukraine*)
Matsumoto K.
(*Yamagata, Japan*)
Mormul P.
(*Warsaw, Poland*)
Mykhailyuik V.
(*Chernivtsi, Ukraine*)
Plachta L.
(*Krakov, Poland*)
Pokas S.
(*Odessa, Ukraine*)
Sabitov I.
(*Moscow, Russia*)
Savchenko O.
(*Kherson, Ukraine*)
Sergeeva A.
(*Odesa, Ukraine*)
Shelekhov A.
(*Tver, Russia*)
Zarichnyi M.
(*Lviv, Ukraine*)

Administrative committee

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Lishchenko N. Dean of faculty of the computer systems and automation ONAFT

Organizing commitee

Cherevko Ye.
Eftekharinasab K.
Fedchenko Yu.
Feshchenko B.
Khohliyk O.
Klishchuk B.
Konovenko N.
Kravchenko A.
Kuznietsova I.
Maksymenko S.
.
Osadchuk E.
Plakosh A.
Prus A.
Sergeeva A.
Soroka Yu.

Entropy and phase transitions in Calabi-Yau space

Tetiana Obikhod
(03028, Institute for Nuclear Research, Prospect Nauki, 47, Kiev, Ukraine)
E-mail: obikhod@kinr.kiev.ua

The evolution of the concept of entropy from the mathematical definition of the theory of probability to the physical definition of entropy in various systems is considered. Statistical interpretation of entropy for a macrostate is characterized by N_i microstates

$$W = \frac{N}{N_1 N_2 \ldots} = \frac{N}{\Pi_i N_i}.$$

The equilibrium state corresponds to the maximum probability, which is proportional to the maximum entropy, and is displayed by the Boltzmann-Planck law

$$S = k \cdot lnW.$$

For large N using the Stirling formula

$$lnN! = NlnN - N$$

and taking into account the degeneracy of the energy states, z_i, we have the following formula for the probability of a system of particles

$$W = \frac{N}{n_1 n_2 \ldots n_r} \Pi_i z_i^{n_i}.$$

Within the framework of the AdS/CFT correspondence, a model for determining the entropy of black holes through the number of microstates is considered. It is known that the entropy of a black hole is determined by the Bekenstein-Hawking formula,

$$S = \frac{A}{4G},$$

where A and G are the surface areas of the black hole and the gravitational constant, respectively. In the framework of superstring and D-brane theory, the concept of entropy has changed due to the presence of the extra-dimensional Calabi-Yau space, which is folded at each point of the usual Minkowski space

$$S = \frac{A_{d+p}}{4G_{d+p}},$$

where p are spatial directions of the space $R^p \times S^{d-1}$, with d – the number of space dimensions transverse to the p-directions. According to Strominger [1], a black hole can be represented as a submanifold of such a Calabi-Yau like a pea in a shell. Depending on the dimension of space, the two-dimensional world surface of the string completely surrounds the two-dimensional sphere, the 3-brane surrounds the three-dimensional sphere, etc. Since the black hole tends to deflate and swell, according to the ideology of flop transformations, a rupture of the Calabi-Yau space occurs. In this case, according to Strominger's calculations, the black hole undergoes a phase transition and transforms into a pointlike particle like a photon,

$$\frac{S_{array}}{S_{string}} \sim \left(\frac{R}{r_H}\right)^{1/(d-3)}.$$

So the array dominates for small horizon radii, and the black string dominates for large horizon radii.

String theory spacetimes with conserved quantum numbers can be black holes, but more commonly they are black p-branes. According to papers [2, 3] black hole entropy

$$S_{BH} = \frac{\Omega_{d-2} r_H^{d-2}}{4G_d}$$

can be described in terms of D-brane theory,

$$S_{BH} = \frac{\Omega_{8-p} r_H^{8-p}}{4G_{10-p}} cosh\beta,$$

where $cosh\beta$ depends on the number of branes. For particular cases, when the number of branes that cover a black hole is determined, we can calculate $d = 4$ entropy in usual four-dimensional space-time. D-brane method for a microscopic accounting for S_{BH} of BPS black holes with macroscopic entropy leads to the formula

$$S_{BH} = 2\pi\sqrt{N_2 N_6 N_5 N_m},$$

(N_i - the numbers of i-branes) which is in agreement with black hole entropy formula. [2].

Using BPS - states of D-branes represented by vector bundles of the type

$$\begin{array}{ccc} Spin(k) & \rightarrow & Spin(k+1) \\ & & \downarrow \\ & & S^k \end{array} \tag{1}$$

it can be shown that for $k = 6$, $Spin(6)$ group is isomorphic to the $SU(4)$ group. Since the group describing black holes is $SU(2,2|4) \sim SU(2,2) \times SU(4)$ ($SU(2,2)$ describes the external degrees of freedom, and $SU(4)$ - the internal ones), the greatest interest is of group $SU(4)$. Then we can work with $Spin$ vector bundles, which present D-branes with the phase transitions between them classified with Grothendieck K-group in the framework of the Clifford algebra formalism. As a result, we obtain a chain of phase transitions of a black hole represented by transitions between topological invariants of vector bundles described by K-groups

$$K(S^6) \rightarrow K(S^4) \rightarrow K(S^2) \rightarrow K(S^0) = Z \quad ,$$

which signal about an equidistant set of energy levels of a point-like particle into which the black hole has passed during the phase transition.

References

[1] Ooguri, Hirosi; Strominger, Andrew; Vafa, Cumrun. *Black hole attractors and the topological string*, Physical Review D. 70 (10): 106007 (2004). arXiv:hep-th/0405146.

[2] Amanda W. Peet. *TASI Lectures on Black Holes in String Theory*, arXiv:hep-th/0008241.

[3] Andy Strominger. *Les Houches Lectures on Black Holes*, arXiv:hep-th/9501071.