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- Geometry and topology of differentiable manifolds
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# Geometric interpretation of first Betti numbers of orbits of smooth functions

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Let  $M$  be a compact connected surface and  $P$  is a real line  $\mathbb{R}$  or a circle  $S^1$ . Denote by  $\mathcal{F}(M, P)$  the space of smooth functions  $f \in C^\infty(M, P)$  satisfying the following conditions:

- 1) the function  $f$  takes constant value at  $\partial M$  and has no critical point in  $\partial M$ ;
- 2) for every critical point  $z$  of  $f$  there is a local presentation  $f_z: \mathbb{R}^2 \rightarrow \mathbb{R}$  of  $f$  near  $z$  such that  $f_z$  is a homogeneous polynomial  $\mathbb{R}^2 \rightarrow \mathbb{R}$  without multiple factors.

Let  $X$  be a closed subset of  $M$ . Denote by  $\mathcal{D}(M, X)$  the group of  $C^\infty$ -diffeomorphisms of  $M$  fixed on  $X$ , that acts on the space of smooth functions  $C^\infty(M, P)$  by the rule:  $(f, h) \mapsto f \circ h$ , where  $h \in D(M, X)$ ,  $f \in C^\infty(M, P)$ .

The subset  $\mathcal{S}(f, X) = \{h \in D(M, X) \mid f \circ h = f\}$  is called the *stabilizer* of  $f$  with respect to the action above and  $\mathcal{O}(f, X) = \{f \circ h \mid h \in D(M, X)\}$  is *orbit* of  $f$ . Denote by  $\mathcal{D}_{id}(M, X)$  the identity path component of  $\mathcal{D}(M, X)$  and let  $\mathcal{S}'(f, X) = \mathcal{S}(f) \cap \mathcal{D}_{id}(M, X)$ .

Homotopy types of stabilizers and orbits of Morse functions were calculated in a series of papers by Sergiy Maksymenko, Bohdan Feshchenko, Elena Kudryavtseva and others. Furthermore, precise algebraic structure of such groups for the case  $M \neq S^2, T^2$  was described in [1]. In particular, the following theorem holds.

**Theorem 1.** [1] *Let  $M$  be a connected compact oriented surface except 2-sphere and 2-torus and let  $f \in \mathcal{F}(M, P)$ . Then  $\pi_0 \mathcal{S}'(f, \partial M) \in \mathcal{B}$ , where  $\mathcal{B}$  is a minimal class of groups satisfying the following conditions:*

- 1)  $1 \in \mathcal{B}$ ;
- 2) if  $A, B \in \mathcal{B}$ , then  $A \times B \in \mathcal{B}$ ;
- 3) if  $A \in \mathcal{B}$  and  $n \geq 1$ , then  $A \wr_n \mathbb{Z} \in \mathcal{B}$ .

Note that a group  $G$  belongs to the class  $\mathcal{B}$  iff  $G$  is obtained from trivial group by a finite number of operations  $\times, \wr_n \mathbb{Z}$ . It is easy to see that every group  $G \in \mathcal{B}$  can be written as a word in the alphabet  $\mathcal{A} = \{1, \mathbb{Z}, (, ), \times, \wr_2, \wr_3, \wr_4, \dots\}$ . We will call such word a *realization* of the group  $G$  in the alphabet  $\mathcal{A}$ .

Denote by  $\beta_1(G)$  the number of symbols  $\mathbb{Z}$  in the realization  $\omega$  of group  $G$ . The number  $\beta_1(G)$  is the rank of the center  $Z(G)$  and the quotient-group  $G/[G, G]$  (Theorem 1.8 [2]). Note, this number depends only on the group  $G$ , not the presentation  $\omega$ . Moreover,  $\beta_1(G)$  is first Betti number of  $\mathcal{O}(f)$ .

Edge of  $\Gamma_f$  will be called *external* if it is incident to the vertex of  $\Gamma_f$  that is corresponding to a non-degenerate critical point of  $f$  or non-fixed boundary component of  $\partial M$  with respect to the action of  $\mathcal{S}'(f, W)$  for  $f$ -adapted submanifold  $X$  which contains  $W = S^1 \times 0$ . Otherwise, it will be called *internal*. Denote by  $\sharp \text{Orb}_{int}(M, W)$  the number of orbits of the action of  $\mathcal{S}'(f, W)$  on internal edges of  $\Gamma_{f|_X}$ .

**Theorem 2.** *Let  $M$  be a disk  $D^2$  or a cylinder  $C = S^1 \times [0, 1]$  and  $f \in \mathcal{F}(M, P)$ . Then*

$$\sharp \text{Orb}_{int}(M, W) = \beta_1(\pi_0 S'(f, \partial M)),$$

*where  $W = \partial M$  if  $M = D^2$  or  $W = S^1 \times 0$  if  $M$  is a cylinder.*

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