

International
Scientific Conference



Algebraic
and Geometric
Methods
of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

SCIENTIFIC COMMITTEE

- | | |
|---|--|
| • Vladimir Balan (<i>Bucharest, Romania</i>) | • Volodymyr Lyubashenko (<i>Kyiv, Ukraine</i>) |
| • Taras Banakh (<i>Lviv, Ukraine</i>) | • Sergiy Maksymenko (<i>Kyiv, Ukraine</i>) |
| • Dmytro Bolotov (<i>Kharkiv, Ukraine</i>) | • Koji Matsumoto (<i>Yamagata, Japan</i>) |
| • Vyacheslav Boyko (<i>Kyiv, Ukraine</i>) | • Piotr Mormul (<i>Warsaw, Poland</i>) |
| • Yulia Fedchenko (<i>Odesa, Ukraine</i>) | • Maryna Nesterenko (<i>Kyiv, Ukraine</i>) |
| • Oleg Gutik (<i>Lviv, Ukraine</i>) | • Roman Popovych (<i>Kyiv, Ukraine</i>) |
| • Olena Karlova (<i>Chernivtsi, Ukraine</i>) | • Alexandr Prishlyak (<i>Kyiv, Ukraine</i>) |
| • Volodymyr Kiosak (<i>Odesa, Ukraine</i>) | • Aleksandr Savchenko (<i>Kherson, Ukraine</i>) |
| • Nadiia Konovenko (<i>Odesa, Ukraine</i>) | |

ORGANIZING COMMITTEE

- | | |
|---|--|
| • Nadiia Konovenko (<i>Odesa, Ukraine</i>) | • Bohdan Mazhar (<i>Kyiv, Ukraine</i>) |
| • Yuliya Fedchenko (<i>Odesa, Ukraine</i>) | • Sergiy Maksymenko (<i>Kyiv, Ukraine</i>) |
| • Mykola Lysynskiy (<i>Kyiv, Ukraine</i>) | • Alexandr Prishlyak (<i>Kyiv, Ukraine</i>) |

Applications of dimension theory to embeddability problems in topological data analysis: the case study of the Gromov-Hausdorff distance

Nicolò Zava

(Institute of Science and Technology Austria, ISTA)

E-mail: nicolo.zava@ist.ac.at

Topological data analysis (TDA) is a recent and fast-growing subject aiming to apply topological techniques to a wide range of applications. Areas where topological data analysis has found applications include neuroscience, image recognition, biology and evolutionary networks. Treating datasets or topologically inspired invariants assigned to them as points of a metric space is a central idea. However, comparing objects in this new metric space is often computationally challenging. Therefore, a technique usually deployed is mapping those objects in a Hilbert space using *vectorisation methods* or *kernels* to allow their implementation in machine learning pipelines. The quest to construct such maps that distort the distances in a controlled way is a crucial research area.

In this talk, we present how dimension theory can fruitfully assist this search. Regarding these maps as examples of certain metric embedding classes is the key-viewpoint shift. As a leading example, we discuss embeddability results for families of metric spaces endowed with the Gromov-Hausdorff distance. In addition to the intrinsic interest of these results given by the importance of this metric in Riemannian geometry and geometric group theory, they also impact TDA since the Gromov-Hausdorff distance was recently proposed as a theoretical framework for shape and dataset comparisons. More precisely, we show the following results:

- the space \mathcal{GH}_n of metric spaces with at most n elements can be coarsely embedded into a Hilbert space;
- their union $\bigcup_{n \in \mathbb{N}} \mathcal{GH}_n$ cannot be coarsely embedded into any Hilbert space;
- the space of metric spaces whose diameter is bounded by r cannot be bi-Lipschitz embedded into any \mathbb{R}^m .

If time permits, we conclude by presenting results concerning the embeddability of persistence diagrams, one of the cornerstone notions of TDA, and periodic point sets, a generalisation of lattices used to represent crystals in material science and pharmaceuticals.

REFERENCES

- [1] N. Zava. *Coarse and bi-Lipschitz embeddability of subspaces of the Gromov-Hausdorff space into Hilbert spaces*, arXiv:2303.04730.

Balayage on locally compact spaces

Natalia Zorii

(Institute of Mathematics of NASU, Tereshchenkivska 3, 02000, Kyiv, Ukraine)

E-mail: zorii@imath.kiev.ua

This talk is based on [1]–[3], and it is devoted to the theory of potentials on a locally compact (Hausdorff) space X with respect to a *kernel* κ , κ being thought of as a symmetric, lower semi-continuous function $\kappa : X \times X \rightarrow [0, \infty]$. To be exact, we are interested in generalizations of the classical theory of balayage (sweeping out) on \mathbb{R}^n , $n \geq 2$ (see e.g. [4]–[7]), to a suitable kernel κ on X .

We denote by \mathfrak{M} the linear space of all (real-valued Radon) measures μ on X , equipped with the *vague* topology of pointwise convergence on the continuous functions $\varphi : X \rightarrow \mathbb{R}$ of compact support, and by \mathfrak{M}^+ the cone of all positive $\mu \in \mathfrak{M}$. (For the theory of measures and integration on X , we refer to Bourbaki [8].) Given $\mu, \nu \in \mathfrak{M}$, the *mutual energy* and the *potential* are introduced by

$$I(\mu, \nu) := \int \kappa(x, y) d(\mu \otimes \nu)(x, y) \quad \text{and} \quad U^\mu(x) := \int \kappa(x, y) d\mu(y), \quad x \in X,$$

respectively, provided the value on the right is well defined as a finite number or $\pm\infty$. For $\mu = \nu$, the mutual energy $I(\mu, \nu)$ defines the *energy* $I(\mu, \mu) =: I(\mu)$ of $\mu \in \mathfrak{M}$.

In what follows, a kernel κ is assumed to satisfy the *energy principle*, which means that $I(\mu) \geq 0$ for all (signed) $\mu \in \mathfrak{M}$, and moreover that $I(\mu) = 0$ only for $\mu = 0$. Then all $\mu \in \mathfrak{M}$ of finite energy form a pre-Hilbert space \mathcal{E} with the inner product $\langle \mu, \nu \rangle := I(\mu, \nu)$ and the energy norm $\|\mu\| := \sqrt{I(\mu)}$, cf. [9, Lemma 3.1.2]. The topology on \mathcal{E} introduced by means of this norm is said to be *strong*.

In addition, we shall always assume that κ satisfies the *consistency principle*, which means that the cone $\mathcal{E}^+ := \mathcal{E} \cap \mathfrak{M}^+$ is *complete* in the induced strong topology, and that the strong topology on \mathcal{E}^+ is *finer* than the vague topology on \mathcal{E}^+ ; such a kernel is said to be *perfect* (Fuglede [9]). Thus any strong Cauchy net $(\mu_j) \subset \mathcal{E}^+$ converges *both strongly and vaguely* to the same unique measure $\mu_0 \in \mathcal{E}^+$.

Yet another permanent requirement on κ is that it satisfies the *domination principle*, which means that for any $\mu \in \mathcal{E}^+$ and any $\nu \in \mathfrak{M}^+$ with $U^\mu \leq U^\nu$ μ -a.e., the same inequality holds on all of X .

For any $A \subset X$, we denote by \mathfrak{C}_A the upward directed set of all compact subsets K of A , where $K_1 \leq K_2$ if and only if $K_1 \subset K_2$. If a net $(x_K)_{K \in \mathfrak{C}_A} \subset Y$ converges to $x_0 \in Y$, Y being a topological space, then we shall indicate this fact by writing: $x_K \rightarrow x_0$ in Y as $K \uparrow A$.

Given $A \subset X$, we denote by \mathfrak{M}_A^+ the class of all $\mu \in \mathfrak{M}^+$ *concentrated on A* , which means that $X \setminus A$ is locally μ -negligible, or equivalently that A is μ -measurable and $\mu = \mu|_A$, $\mu|_A$ being the trace of μ to A . Also write $\mathcal{E}_A^+ := \mathfrak{M}_A^+ \cap \mathcal{E}$, and let \mathcal{E}'_A stand for the closure of \mathcal{E}_A^+ in the strong topology on \mathcal{E} . Being a strongly closed subcone of the strongly complete cone \mathcal{E}^+ , \mathcal{E}'_A is likewise strongly complete.

Denoting by $c_*(E)$ and $c^*(E)$ the *inner* and *outer* capacity of $E \subset X$, respectively [9, Section 2.3], we say that an assertion $\mathcal{A}(x)$ involving a variable point $x \in X$, holds *nearly everywhere* (*n.e.*), resp. *quasi-everywhere* (*q.e.*), on a set A if $c_*(E) = 0$, resp. $c^*(E) = 0$, where $E := \{x \in A : \mathcal{A}(x) \text{ fails}\}$.

Theorem 1. *For any $A \subset X$ and any $\sigma \in \mathcal{E}^+$, there exists $\sigma^A \in \mathcal{E}'_A$, called the inner balayage of σ to A , that is uniquely characterized by any one of the following (equivalent) assertions.*

(i) σ^A is the unique measure in \mathcal{E}'_A having the property

$$U^{\sigma^A} = U^\sigma \quad \text{n.e. on } A.$$

(ii) σ^A is the unique orthogonal projection of σ in the pre-Hilbert space \mathcal{E} onto the (convex, strongly complete) cone \mathcal{E}'_A , that is, $\sigma^A \in \mathcal{E}'_A$ and

$$\|\sigma - \sigma^A\| = \min_{\mu \in \mathcal{E}'_A} \|\sigma - \mu\|.$$

(iii) σ^A is the unique measure in \mathcal{E}^+ satisfying any one of the following three limit relations:

$$\begin{aligned} \sigma^K &\rightarrow \sigma^A && \text{strongly in } \mathcal{E}^+ \text{ as } K \uparrow A, \\ \sigma^K &\rightarrow \sigma^A && \text{vaguely in } \mathcal{E}^+ \text{ as } K \uparrow A, \end{aligned}$$

$$U^{\sigma^K} \uparrow U^{\sigma^A} \quad \text{pointwise on } X \text{ as } K \uparrow A,$$

where σ^K denotes the only measure in \mathcal{E}_K^+ with the property $U^{\sigma^K} = U^\sigma$ n.e. on K .

(iv) σ^A is the only measure in the class $\Gamma_{A,\sigma}$ having the property

$$U^{\sigma^A} = \min_{\mu \in \Gamma_{A,\sigma}} U^\mu \quad \text{on all of } X, \quad (1)$$

where $\Gamma_{A,\sigma} := \{\mu \in \mathcal{E}^+ : U^\mu \geq U^\sigma \text{ n.e. on } A\}$.

(v) σ^A is the only measure in the class $\Gamma_{A,\sigma}$, introduced above, with the property

$$\|\sigma^A\| = \min_{\mu \in \Gamma_{A,\sigma}} \|\mu\|.$$

Theorem 2. *If a space X is second-countable, while a set A is Borel, then Theorem 1 remains valid with "n.e. on A " replaced throughout by "q.e. on A ". The measure ω^{*A} , thereby uniquely determined, is said to be the outer balayage of ω onto A . Actually, $\omega^{*A} = \omega^A$. (Compare with [10, Theorem 4.12].)*

Remark 3. All the above-mentioned assumptions on a space X and a kernel κ are fulfilled by:

- ✓ The α -Riesz kernels $|x-y|^{\alpha-n}$ of order $\alpha \in (0, 2]$, $\alpha < n$, on \mathbb{R}^n , $n \geq 2$ (see [6, Theorems 1.15, 1.18, 1.27, 1.29]).
- ✓ The associated α -Green kernels, where $\alpha \in (0, 2]$ and $\alpha < n$, on an arbitrary open subset of \mathbb{R}^n , $n \geq 2$ (see [11, Theorems 4.6, 4.9, 4.11]).
- ✓ The (2-)Green kernel on a planar Greenian set (see [5] and [7, Sections I.V.10, I.XIII.7]).

(We emphasize that some of the results formulated above are new even for these classical kernels.)

Remark 4. The theory of balayage thereby developed has already been shown to be a powerful tool in the well-known Gauss variational problem, see [12].

Problem 5. What kind of additional assumptions on X and κ would make it possible to generalize the theory of balayage, presented above, to Radon measures on X of *infinite* energy?

REFERENCES

- [1] Natalia Zorii. Balayage of measures on a locally compact space. *Analysis Mathematica*, 48(1) : 249–277, 2022.
- [2] Natalia Zorii. On the theory of capacities on locally compact spaces and its interaction with the theory of balayage. *Potential Analysis*, 59(3) : 1345–1379, 2023.
- [3] Natalia Zorii. On the theory of balayage on locally compact spaces. *Potential Analysis*, 59(4) : 1727–1744, 2023.
- [4] Henri Cartan. Théorie générale du balayage en potentiel newtonien. *Annales de l'université de Grenoble*, 22 : 221–280, 1946.
- [5] Ralph E. Edwards. Cartan's balayage theory for hyperbolic Riemann surfaces. *Annales de l'institut Fourier*, 8 : 263–272, 1958.
- [6] Naum S. Landkof. *Foundations of Modern Potential Theory*. Berlin : Springer, 1972.
- [7] Joseph L. Doob. *Classical Potential Theory and Its Probabilistic Counterpart*. Berlin : Springer, 1984.
- [8] Nicolas Bourbaki. *Integration. Chapters 1–6*. Berlin : Springer, 2004.
- [9] Bent Fuglede. On the theory of potentials in locally compact spaces. *Acta Mathematica*, 103 (3–4) : 139–215, 1960.
- [10] Bent Fuglede. Symmetric function kernels and sweeping of measures. *Analysis Mathematica*, 42(3) : 225–259, 2016.
- [11] Bent Fuglede, Natalia Zorii. Green kernels associated with Riesz kernels. *Annales Fennici Mathematici*, 43(1) : 121–145, 2018.
- [12] Natalia Zorii. Minimum energy problems with external fields on locally compact spaces. *Constructive Approximation*, 59(2) : 385–417, 2024.

M. Hrechnieva, P. Stiehintseva <i>On the type of Grassman image of a time-like minimal surface in Minkowski space</i>	120
L. Bunimovich, Y. Su <i>Open billiards, chaos and limit theorems</i>	121
E. Sevost'yanov, V. Targonskii <i>On the inverse Poletsky inequality with a cotangent dilatation</i>	121
H. Tashiro <i>Hasse norm theorem for 3-manifolds</i>	123
T. T. Truong <i>A new Newton-type method and connections to Schroder theorem, Voronoi's diagrams, Newton's flows and the Riemann hypothesis</i>	124
O. Vinnichenko, V. Boyko, R. Popovych <i>Geometric and algebraic properties of dispersionless Nizhnik equation</i>	125
I. Vlasenko <i>Chain-regular and regular components of the wandering set of surface homeomorphisms</i>	127
C. Vural, E. Demir <i>Dynamics of influenza with the rates of vaccination and treatment</i>	128
M. Watari <i>Topology of the Hilbert Schemes of monomial plane curve singularities</i>	128
D. Zashkolnyi <i>Self-similar actions of the fundamental group of the Klein bottle</i>	130
N. Zava <i>Applications of dimension theory to embeddability problems in topological data analysis: the case study of the Gromov-Hausdorff distance</i>	131
N. Zorii <i>Balayage on locally compact spaces</i>	131
О. Дажук, І. Курбатова, О. Яблокова <i>Узагальнені аналоги теореми Яно-Вестлейка</i>	134
В. Кіосак, О. Латиш <i>Геодезичні відображення псевдоріманових просторів</i>	135
О. Лесечко, О. Савченко <i>Спеціальні келерові простори</i>	137
О. Назаренко, В. Думанська <i>Відображення келерових просторів</i>	138
В. Петров, О. Василів <i>Метод растрової візуалізації перетинаючих геометричних тіл та побудови розгортки</i>	139
Т. Подоусова, Ю. Федченко, Н. Вашпанова <i>Ундулоїди та деякі їх деформації</i>	141
О. Яблокова, І. Курбатова, О. Дажук <i>Канонічні F-планарні відображення</i>	142