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Algebraic and Geometric Methods of Analysis

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List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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O-spheroids in metric and linear normed spaces

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Definition 1. Open O-spheroid with rank n, or O-spheroid with rank n, in a metric space (X, ρ) with a metric ρ, $n \in \mathbb{N}$, is a set

$$A = \{x \in X \mid \rho(x, x_1) + \cdots + \rho(x, x_n) < a\},$$

where $x_1, \ldots, x_n$ are different fixed points of the space (X, ρ), called the foci, and a is a fixed positive number, called the distance. We can get a respective definition in linear normed spaces.

Definition 2. Closed O-spheroid with rank n in a metric space (X, ρ) with a metric ρ, $n \in \mathbb{N}$, is a set

$$A = \{x \in X \mid \rho(x, x_1) + \cdots + \rho(x, x_n) \le a\},$$

where $x_1, \ldots, x_n$ are different fixed points of the space (X, ρ), called the foci, and a is a fixed positive number, called the distance. We can get a respective definition in linear normed spaces.

Remark 3. $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is an open O-spheroid with rank n with the foci in points $x_1, \ldots, x_n$ and the distance a. If we talk about open O-spheroid understanding what namely O-spheroid we discuss, we note it $\mathbb{S}_n$.

Definition 4 ([11, c. 193])**.** Border of (open or closed) O-spheroid with rank n, or n-ellipse with the foci $x_1, \ldots, x_n$ and the distance a, in a metric space (X, ρ) we name the set

$$A = \{x \in X \mid \rho(x, x_1) + \cdots + \rho(x, x_n) = a\}.$$

Definition 5. Focal closeness of our O-spheroid with rank n equals to

$$\pi(\mathbb{S}_n(x_1, \ldots, x_n; a)) := \min_{1 \le i < j \le n} \rho(x_i, x_j).$$

Definition 6. Focal remoteness of our O-spheroid with rank n equals to

$$\Phi(\mathbb{S}_n(x_1, \ldots, x_n; a)) := \max_{1 \le i < j \le n} \rho(x_i, x_j).$$

Definition 7. If all the foci belong to O-spheroid, then it is called a *multicentered* one.

Theorem 8. *Let's assume we have an O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ in a metric space (X, ρ) with a metric ρ, $n > 1$. If it is multicentered then*

$$\pi(\mathbb{S}_n) < \frac{a}{n-1}.$$

Theorem 9. *Let's assume we have an O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ in a metric space (X, ρ) with a metric ρ, $n > 1$. If we have that*

$$\Phi(\mathbb{S}_n) < \frac{a}{n-1},$$

then this O-spheroid is multicentered.

Theorem 10. *Either all open and closed O-spheroids in arbitrary metric space (X, ρ) with a metric ρ, or their borders, are bounded sets.*

Remark 11. All closed O-spheroids in any Euclidean metric space $(\mathbb{R}^m, \rho)$ with a standard metric ρ are compact sets.

Definition 12. Metric space (X, ρ) with a metric ρ is called *convex*, if next conditions are satisfied:

1) X is a linear vector space;
2) $\forall \{x, y, z\} \subset X \; \forall \alpha \in [0; 1]$ we get:

$$\rho(\alpha x + (1 - \alpha)y, z) \leq \alpha\rho(x, z) + (1 - \alpha)\rho(y, z).$$

Theorem 13. *If (X, ρ) is a convex metric space with a metric ρ, then $\forall \{x_1, \ldots, x_n\} \subset X \; \forall a > 0$ open O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a connected set.*

Remark 14. All O-spheroids in linear normed spaces are connected sets.

Theorem 15. *If (X, ρ) is a convex metric space with a metric ρ, then $\forall \{x_1, \ldots, x_n\} \subset X \; \forall a > 0$ open O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a connected set.*

Theorem 16. *Let's assume that $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a non-empty O-spheroid in a convex metric space (X, ρ) with a metric ρ. Then its border is equal to its boundary.*

Definition 17 ([7, c. 236])**.** *Fermat–Torricelli point* for fixed points $\{x_1, \ldots, x_n\}$ is such point $\overline{x} \in X$, that $\forall x \in X$:

$$\sum_{k=1}^{n} \rho(\overline{x}, x_k) \leq \sum_{k=1}^{n} \rho(x, x_k).$$

Definition 18. *Voronoi radius* of O-spheroid $\mathbb{S}_n(x_1, \ldots, x_n; a)$ we call number

$$R(\mathbb{S}_n) := \sup_{x \in \mathbb{S}_n} \inf_{y \in \partial\mathbb{S}_n} \rho(x, y).$$

Theorem 19. *Let's assume that $\mathbb{S}_n(x_1, \ldots, x_n; a)$ is a non-empty O-spheroid in any Euclidean metric space $(\mathbb{R}^m, \rho)$ with a standard metric ρ, meanwhile $\overline{x}$ is a Fermat–Torricelli point for its foci. Then next inequality is correct:*

$$\frac{a - \sum\limits_{k=1}^{n} \rho(\overline{x}, x_k)}{n} \leq R(\mathbb{S}_n).$$

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