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# Functions with three critical points on closed non-oriented 3-manifolds

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Let  $M$  be a closed smooth 3-manifold and  $f, g : M \rightarrow \mathbb{R}$  be smooth functions.

**Definition 1.** Functions  $f, g : M \rightarrow \mathbb{R}$  are called *topologically equivalent* if there are homeomorphisms  $h : M \rightarrow M$  and  $k : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f \circ h = k \circ g$ . If  $k$  additionally preserve the orientation of  $\mathbb{R}$ ,  $f$  and  $g$  are called *topologically conjugated* and homeomorphisms  $k, g$  by *conjugated*.

The problem of topological classification of Morse functions was solved in [1] and [2] for closed manifolds of different dimensions. The same result for arbitrary functions with isolated critical point on closed 2-manifolds was obtained in [3]. The relevance of this problem is contributed by the close connection with the Hamiltonian dynamical system's classification in dimensions 2 and 4. Local topological classification with isolated critical points and global topological classification with 3 critical points on oriented manifold were obtained in [4] The main issued of this research is to get similar results in non-oriented case.

It is known [3] that if  $p$  is an isolated critical point,  $y = f(p)$ , then there exists closed neighborhood  $U(p)$  such that

$$f^{-1}(y) \cap U(p) = \text{Con}(\cup S_i^1).$$

Here  $\text{Con}(\cup S_i^1)$  is a cone on a disjoint union of circles  $S_i^1$ , that is the union of two-dimensional disks, the centers of which are pasted together.

In order to describe the behavior of function in a neighborhood of critical point  $p$  we will construct a tree (graph without cycles)  $Gf_p$ . Let  $U(p)$  be the neighborhood described above, which boundary is the sphere  $S^2$  and  $\partial(f^{-1}(y) \cap U(p)) = \cup S_i^1$  is the union of the embedded circles. To each component  $D_j$  of  $S^2 \setminus \cup S_i^1$  we put in correspondence vertex  $v_j$  of the graph  $Gf_p$  and to each circle  $S_i^1$  we put an edge  $e_i$ . The vertex  $v_j$  is incident to  $e_i$  if the boundary of  $D_j$  contains  $S_i^1$ . Thus,  $v_i$  and  $v_j$  are connected by an edge if  $D_i$  and  $D_j$  are neighbor.

**Theorem 2.** *Let  $p$  and  $q$  be isolated critical points of smooth functions  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^1$  and  $g : \mathbb{R}^3 \rightarrow \mathbb{R}^1$  correspondingly. Then there are neighborhoods  $U$  of  $p$  and  $V$  of  $q$  and homeomorphisms  $h : U \rightarrow V$  and  $k : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f \circ h = k \circ g$  if and only if graphs  $Gf_p$  and  $Gg_q$  are isomorphic.*

We construct a distinguishing graph  $Gf$  for the function  $f$  with 3 critical points on 3-manifold, such that it has the following properties:

1) The vertices of the graph are divided into four types: white, black, gray and non-colored. The number of vertices of each color (the first three types) is same. The non-colored vertices have degree 3. Each white vertex is equipped with the orientation number (+1 or -1).

2) If from the graph we remove vertices of one color and edges that incident to them, we obtain simply-connected graphs (tree)  $Gf'_i$ .

**Theorem 3.** *Let  $f, g : M \rightarrow \mathbb{R}$  be smooth functions that have three critical points on a smooth closed 3-manifold. The functions  $f$  and  $g$  are conjugated if and only if their distinguishing graphs are equivalent.*

**Example 4.** The number of

- (1) topologically non-equivalent and topologically non-conjugated functions defined on  $S^1 \tilde{\times} S^2$  equals 1;
- (2) topologically non-equivalent functions with three critical points on  $S^1 \tilde{\times} S^2 \sharp S^1 \tilde{\times} S^2$  equals 16;
- (3) topologically non-conjugated functions defined on  $S^1 \tilde{\times} S^2 \sharp S^1 \tilde{\times} S^2$  equals 24.

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