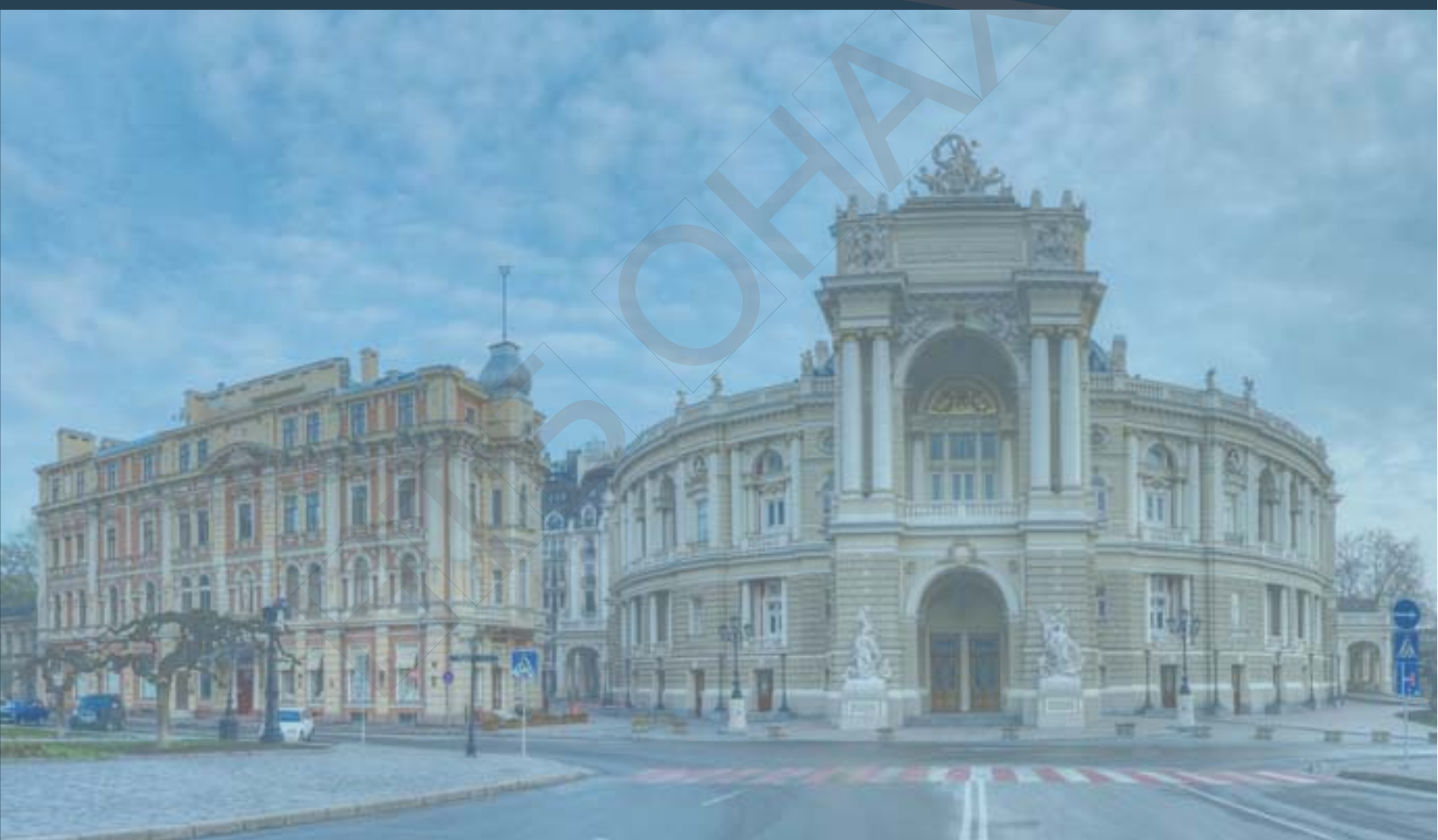


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**“Algebraic and Geometric
Methods of Analysis”**

Book of abstracts



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ФІТБ ОНАФТ

Quantum calculus and singularities of quasi-discriminant sets

Alexander Batkhin

(Keldysh Institute of Applied Mathematics of RAS (Moscow) & Moscow Institute of Physics and Technology (Dolgoprudny), Russia)
E-mail: batkhin@gmail.com

Let $g : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto g(x)$ be a given smooth one-to-one map of the real axis, which is the domain of polynomial $f(x)$ with arbitrary coefficients. We want to find conditions on the coefficients of the polynomial under which it has at least a pair of roots t_i, t_j satisfying the relation $g(t_i) = t_j$ and investigate the structure of the algebraic variety in the space of coefficients possessing such property.

Here we consider a generalization of the classical discriminant of the polynomial. This generalization naturally includes the classical discriminant and its analogs emerging when the q -differential and difference operators are used. The aim of this research is to propose an efficient algorithm for calculating the parametric representation of all components of the g -discriminant set $\mathcal{D}_g(f)$ of the monic polynomial $f_n(x)$ of degree n .

Define the q -bracket $[a]_q$, q -Pochhammer symbol $(a; q)_n$, q -factorial $[n]_q!$, q -binomial coefficients (Gaussian) coefficients $\begin{bmatrix} n \\ k \end{bmatrix}_q$, g -binomial $\{x; t\}_{n;g}$ as follows: $[a]_q = \frac{q^a - 1}{q - 1}$, $(a; q)_n = \prod_{k=0}^{n-1} (1 - aq^k)$, $(a; q)_0 = 1$, $[n]_q! = \prod_{k=1}^n [k]_q = \frac{(q; q)_n}{(1 - q)^n}$, $q \neq 1$, $\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q! [k]_q!} = \prod_{i=1}^k \frac{q^{n-i+1} - 1}{q^i - 1}$, $\{x; t\}_{n;g} \equiv \prod_{i=0}^{n-1} (x - g^i(t))$, $\{x; t\}_{0;q} = 1$. Here g^k is the k -th iteration of the diffeomorphism g , $k \in \mathbb{Z}$. As $q \rightarrow 1$, all these objects become classical.

Let $f_n(x)$ be a monic polynomial of degree n with complex coefficients defined by $f_n(x) \stackrel{\text{def}}{=} x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_n$. Let \mathbb{P} be the space of polynomials over \mathbb{R} and let $g : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto qx + \omega$, $q, \omega \in \mathbb{R}$, $q \neq \{-1, 0\}$, be a linear diffeomorphism on \mathbb{R} that induces a linear **Hahn operator** \mathcal{A}_g on \mathbb{P} , satisfying the following two conditions: (1) the degree reduction: $\deg(\mathcal{A}_g f_n)(x) = n - 1$; in particular, $\mathcal{A}_g x = 1$; (2) Leibnitz rule analogue: $(\mathcal{A}_g x f_n)(x) = f_n(x) + g(x)(\mathcal{A}_g f_n)(x)$.

The Hahn operator \mathcal{A}_g called below **g -derivative** has the form

$$(\mathcal{A}_g f)(x) \stackrel{\text{def}}{=} \begin{cases} \frac{f(qx + \omega) - f(x)}{(q - 1)x + \omega}, & x \neq \omega_0, \\ f'(\omega_0), & x = \omega_0, \end{cases}$$

where $\omega_0 = \omega/(1 - q)$ is the fixed point of g . Parameters q and ω are satisfied the following conditions $q, \omega \in \mathbb{R}$, $q \neq \{-1, 0\}$ and $(q, \omega) \neq (1, 0)$. The g -derivative \mathcal{A}_g can be considered as a generalization of the q -differential Jackson operator \mathcal{A}_q at $\omega = 0$, $q \neq 1$, as the difference operator Δ_ω at $q = 1$ and as the classical derivative d/dx in the limit $q \rightarrow 1$ and $\omega = 0$.

The q -calculus has become a part of the more general construct called quantum calculus [1, 2]. It has numerous applications in various fields of modern mathematics and theoretical physics. The pair of roots t_i, t_j , $i, j = 1, \dots, n$, $i \neq j$ of the polynomial $f_n(x)$ is called **g -coupled** if $g(t_i) = t_j$.

Problem 1. In the coefficient space $\Pi \equiv \mathbb{C}^n$ of the polynomial $f_n(x)$, investigate the g -discriminant set denoted $\mathcal{D}_g(f_n)$ on which this polynomial has at least one pair of g -coupled roots.

The sequence $\text{Seq}_g^{(k)}(t_1)$ of g -coupled roots of length k is defined as the finite sequence $\{t_i\}$, $i = 1, \dots, k$ in which each term, beginning with the second one, is a g -coupled root of the preceding term: $g(t_i) = t_{i+1}$. The initial root t_1 is called the generating root of the sequence $\text{Seq}_g^{(k)}(t_1)$.

For each fixed set of parameters q, ω , the g -discriminant set $\mathcal{D}_g(f_n)$ consists of a finite set of varieties \mathcal{V}_k on each of which $f_n(x)$ has k sequences $\text{Seq}_g^{(l_i)}(t_i)$ of g -coupled roots of length i with different generating roots t_i , $i = 1, \dots, k$. To obtain an expression for the generalized (sub)discriminant of the polynomial $f_n(x)$ in terms of its coefficients, any method available in the classical elimination theory can be used. If we replace the derivative $f'_n(x)$ by the polynomial $\mathcal{A}_g f_n(x)$, then any matrix method for calculating the resultant of a pair of polynomials gives an expression of the generalized k -th subdiscriminant $D_g^{(k)}(f_n)$ [3].

Theorem 2. *The polynomial $f_n(x)$ has exactly $n-d$ different sequences of g -coupled roots, iff the first nonzero element in the sequence of i -th generalized subdiscriminants $D_g^{(i)}(f_n)$ is the subdiscriminant $D_g^{(d)}(f_n)$ with the index d .*

Consider the partition $\lambda = [1^{n_1} 2^{n_2} 3^{n_3} \dots]$ of a natural number n . Every partition λ of n determines the structure of the g -coupled roots of the polynomial $f_n(x)$, and this structure is associated with the algebraic variety \mathcal{V}_l^i , $i = 1, \dots, p_l(n)$ of dimension l corresponding to the number of different generating roots t_i in the coefficient space Π . The partition $[n^1]$ corresponding to the case when there is a unique sequence of roots of length n specified by the generating root t_1 . Then, the polynomial $f_n(x)$ is a g -binomial $\{x; t_1\}_{n;g}$ and its coefficients a_i can be represented in terms of the elementary symmetric polynomials $\sigma_i(x_1, x_2, \dots, x_n)$ calculated on the roots $g^j(t_1)$, $j = 0, \dots, n-1$, $a_i = (-1)^i \sigma_i(t_1, g(t_1), \dots, g^{n-1}(t_1))$, $i = 1, \dots, n$.

Theorem 3 ([4]). *Let there be a variety \mathcal{V}_l , $\dim \mathcal{V}_l = l$ on which the polynomial $f_n(x)$ has different sequences of g -coupled roots and the sequence of roots $\text{Seq}_g^{(m)}(t_1)$ has length $m > 1$. The roots of the other sequences are not g -coupled with all roots of the sequence $\text{Seq}_g^{(m)}(t_1)$. Let $\mathbf{r}_l(t_1, \dots, t_l)$ be a parameterization of the variety \mathcal{V}_l . Then for $0 < k < n$, the formula*

$$\mathbf{r}_l(t_1, \dots, t_l, t_{l+1}) = \mathbf{r}_l(t_1, \dots, t_l) + \sum_{i=1}^k \begin{bmatrix} k \\ i \end{bmatrix}_q \frac{[m-i]_q!}{[m]_q!} (\mathcal{A}_g^i \mathbf{r}_l)(t_1) \{t_{l+1}; t_1\}_{i;g} \quad (1)$$

specifies a polynomial parameterization of the part of \mathcal{V}_{l+1} on which there are two sequences of roots $\text{Seq}_g^{(m-k)}(g^k(t_1))$ and $\text{Seq}_g^{(k)}(g(t_{l+1}))$, and the other sequences of roots are the same as on the original variety \mathcal{V}_l .

The structure of singular points of each variety \mathcal{V}_{l+1} can be described in terms of varieties \mathcal{V}_l , connected with it by (1).

The same results on the structure and parametrization of the g -discriminant set $\mathcal{D}_g(f_n)$ can be obtained for other variants of g -derivative, e.g. for the case of Hahn symmetric derivative [2]

$$\tilde{\mathcal{A}}_{q,\omega} f(t) = \frac{f(qt + \omega) - f(q^{-1}(t - \omega))}{(q - q^{-1})t + (1 + q^{-1})\omega}$$

as well.

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