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"Algebraic and Geometric Methods of Analysis"

Book of abstracts



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Algebraic-geometric aspects of function field analogues to abelian varieties

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This communication is a continuation of [6, 7, 8, 9].

Let p be a prime number, $q = p^n$, \mathbb{F}_q be the field with q elements and characteristic p, \mathbb{F} be a finite field extension of a finite field \mathbb{F}_q .

We extend the case of algebraic number fields [6, 7] to the case of function fields in characteristic p > 0 and construct function field analogues to abelian varieties of elliptic and hyperelliptic curves appeared in [8, 9]. In the last case we investigate function field analogues to abelian varieties which are Jacobian varieties of hyperelliptic curves in characteristic p > 0. Recall that for hyperelliptic curves the function field analogues to abelian varieties are function field analogues to Jacobian varieties of the curves. For Jacobians it is possible to define corresponding p-divisible groups. We plan to present results on function field analogues to p-divisible groups of the Jacobian varieties.

Moduli and estimates for hyperelliptic curves of genus $g \geq 2$ over \mathbb{F}_p . Let

$$C: y^2 = f(x)$$

be an algebraic curve and let Disk(C) be the discriminant of f(x). Consider hyperelliptic curve of genus $g \ge 2$ over prime finite field \mathbf{F}_p

$$C_q: y^2 = f(x), \ D(f) \neq 0.$$

For projective closure of C_g the quasiprojective variety

$$S_{g,p} = \{ \mathbf{P}^{2g+2}(\mathbf{F}_p) \setminus (Disk(C_g) = 0) \}$$

parametrizes all hyperelliptic curves of genus g over \mathbf{F}_p . By well known Weil bound (affine case)

$$|\#C_g(\mathbf{F}_p) - p| \le 2g\sqrt{p}.$$

where #C is the number of points on the curve C over ground field. As we can see from Weil (and some more strong) bounds, for $p \geq 17$ any hyperelliptic curve of genus g = 2 has points in \mathbb{F}_p for these prime p. Also for g = 3 every hyperelliptic (h) curve of genus 3 has points in \mathbb{F}_p for $p \geq 37$. For p = 2, 3, 5, 7, 11 there are examples of h-curves of genus 2 that have not points in \mathbb{F}_p . By author's computations any h-curve of genus 2 over \mathbb{F}_{13} has points in the field. Similarly, for p = 2, 3, 5, 7, 11, 13, 17 there are examples of h-curves of genus 3 that have not points in \mathbb{F}_p .

Theorem 1. [8]. Let $p \equiv 3 \mod 4$. Under $p \ge 11$ there is such $a \in \mathbb{F}_p$ that the equation

$$y^2 = x^{\frac{p-1}{2}} + a$$

has no solutions in \mathbb{F}_p .

Global \mathfrak{G} -shtukas and local \mathbb{P} -shtukas [1, 2, 3, 4].

Definition 2. (Hartl, Rad [1, 2]) Let C be a smooth projective geometrically irreducible curve over \mathbb{F}_q . A global \mathfrak{G} -shtuka $\overline{\mathcal{G}}$ over an \mathbb{F}_q -scheme S is a tuple $(\mathcal{G}, s_1, \ldots, s_n, \tau)$ consisting of a \mathfrak{G} -torsor \mathcal{G} over $C_S := C \times_{\mathbb{F}_q} S$, an n-tuple of (characteristic) sections $(s_1, \ldots, s_n) \in C^n(S)$ and a Frobenius connection τ defined outside the graphs of the sections s_i .

For Jacobian varieties it is possible to define corresponding p-divisible groups and their function field analogues.

Definition 3. (Hartl, Rad [1]) Let \mathbb{P} be a flat affine group scheme of finite type over $Spec \ \mathbb{F}[[z]]$ and \mathfrak{G} is a flat affine group scheme of finite type over a smooth projective geometrically irreducible curve over \mathbb{F}_q

Recall that local \mathbb{P} -shtukas are the functional field analogs of p-divisible groups with additional structure and moduli stacks of global \mathfrak{G} -shtukas are the functional field analogs for Shimura varieties. In some cases \mathbb{P} is a paraholic Bruhat-Tits group scheme by Pappas, Rapoport [5] and \mathfrak{G} is a parahoric Bruhat-Tits group scheme over a smooth projective curve over finite field \mathbb{F}_q with q elements of characteristic p. Investigations by U. Hartl [3], by Hartl, Arasteh Rad [1, 2], by U. Hartl, E. Viehmann [4] continue works of V. G. Drinfeld , L. Lafforgue, G. Faltings.

If will sufficient time we plan to give a short review of history of these research.

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Зміст